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Wave Scattering by a Circular Elastic Plate in Water of Finite Depth: A Closed Form Solution

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ABSTRACT

We present a solution for a circular thin plate of shallow draft on water of finite depth subject to linear wave forcing of a single frequency. The solution, which is given in a closed form, is based on decomposing the solution into angular eigenfunctions. The coefficients in the expansion are then found by matching the potential and its derivative at the plate edge and imposing the free edge conditions for the plate. The matching is accomplished by taking the inner product with respect to the vertical eigenfunctions for the free surface. The equations that are derived are transformed so that the final system of equations involves only the unknowns under the plate. Solutions are presented and compared to the results of Meylan (2002), who presented a solution for a plate of arbitrary geometry.

INTRODUCTION

The problem of a linear, floating thin plate of shallow draft subject to wave forcing is a standard problem in hydroelasticity which can be used to model a range of physical systems. In 2 dimensions, many different solution methods exist. For shallow water, a solution was presented by Stoker (1958); for finite depth, solutions have been presented by Meylan and Squire (1994) and Newman (1994), amongst others. In 3 dimensions, a solution for a circular plate on shallow water was presented by Zilman and Miloh (2000), and general solution methods have been presented by Meylan and Squire (1994), Meylan (2002) and Kashiwagi (1998), amongst others. However, with the exception of Zilman and Miloh (2000), these solutions were based on the free-surface Green's function and were highly numerical. Even the solution presented by Meylan and Squire (1996) for a circular plate only exploited the circular geometry to calculate the modes of vibration of the free plate.

The only 3-dimensional solution for a thin plate of shallow draft that is not based on a highly numerical method was presented by Zilman and Miloh (2000). The exact same solution method was also independently derived by Tsubogo (2001). Their solution was for the case of shallow water and a circular plate. The circular geometry allows separation of variables in the angular direction, so that the solution may be found by decoupling the solutions for each angular eigenfunction. Once this has been accomplished, the solution for each angular direction can be found by solving a linear system of 4 equations. These were derived by matching the potential and its derivative, and by imposing the 2 boundary conditions at the edge of the plate.

We present here an extension of the method of Zilman and Miloh (2000) to the case where the water depth is finite. In this case we can still solve for each angular eigenfunction separately, and we match the potential and its derivative and impose the boundary conditions at the plate edge. However, we must match the potential not at a point but throughout the water depth. This matching is accomplished by taking the inner product with respect to the vertical eigenfunctions which satisfy the free surface condition. We present results for the method, which are compared to the results of Meylan (2002).

GOVERNING EQUATIONS

We begin with the equations for the plate-water system in nondimensional form as the problem is so well known. The derivation and nondimensionalisation are discussed in detail in Meylan (2002). We nondimensionalise the spatial variables with respect to a length parameter L (for example, L may be derived from the area of the plate, or L may be the characteristic length $(D/\rho g)^{1/4}$, where D is the rigidity constant of the plate, ρ the density of the water, and g the gravitational constant) and the time variables with respect to $\sqrt{L/g}$. We assume that all motions are time-harmonic with radian frequency $\omega = \sqrt{\alpha}$, so that the velocity potential of the water, $\bar{\phi}(x, t)$, can be expressed as the real part of a complex quantity ϕ:

$$\bar{\phi}(\mathbf{x}, t) = \mathrm{Re}\left\{\phi(\mathbf{x}) e^{-\mathrm{i}\sqrt{\alpha}t}\right\} \tag{1}$$

We will use a cylindrical coordinate system, $\mathbf{x} = (r, \theta, z)$, assumed to have its origin at the centre of the circular plate with radius a. The water is assumed to have constant finite depth H, and the z-direction points vertically upward with the water surface at $z = 0$ and the seafloor at $z = -H$. The boundary value problem can thus

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