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Dynamic Analysis of Loop Formation in Cables Under Compression

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ABSTRACT

This paper contributes a dynamic model for cable loop formation that has previously been studied using static (equilibrium) analyses alone. The cable is modeled as a 1-dimensional elastic continuum that accounts for tension, 2-axis bending and torsion. External forces include weight, buoyancy, fluid drag and inertia. The 3-dimensional shape of the cable is predicted, as time evolves, by a numerical algorithm based on separate finite differencing in time and in space. The dynamics of loop formation and loop release are analyzed for different types of supports, including simple (spherical) and fixed supports.

INTRODUCTION

Cable payout systems are widely used to deploy cables from a surface ship onto the ocean floor. As the cable is spooled, small residual torsion in the cable is unavoidable. At the peel point, the residual torsion has a minor effect on the dynamics of the cable because of the large tensions that develop when the cable is paid out. However, when the cable touches the seabed, the tension is drastically reduced, torsion becomes significant, and it can ultimately dictate the dynamics of the cable. For example, under torsion, a low-tension cable can form a loop. This phenomenon is sometimes referred to as hockling. Loops must be avoided because they may ultimately kink the cable or otherwise degrade its performance; for example, a tight loop in a telecommunication cable may block signal transmission.

Previous research has used simplified models to study some aspects of the loop formation process. In his treatise, Love (1944) presents the classical rod theory, which includes results from previous studies conducted by Clebsch and Kirchhoff. He describes the 3-dimensional deformation of a rod by using results from the differential geometry of a space curve, to which a twist is superimposed. Lu and Perkins (1994) analyze the 3-dimensional equilibria and stability of a low-tension cable submitted to a uniaxial torque. The dynamics of the cable is ignored, and closed-form solutions that describe the 3-dimensional equilibrium states are derived. Similar studies of static loop formation can be found, for example, in the work of Tan and Witz (1992), Rosenthal (1976) and Coyne (1990). These studies all focus on determining the periodic solutions of the nonlinear boundary value problems describing cable equilibrium. Spatially complex solutions have also been determined recently (Thompson and Champneys, 1996; Gottlieb and Perkins, 1999).

Hover (1997) introduces dynamic terms in the equations of motion of the cable, but neglects the effects of cable and fluid inertia. Drag forces are modeled using Morison's formulation, and Euler parameters are used to describe an arbitrary 3-dimensional configuration. The equations of motion are discretized using a box method, and Newton-Raphson iteration is applied to find the solution to the resulting algebraic equations. The author studies the evolution of the shape of an asymmetric catenary subjected to a prescribed angle of twist at its upper end.

Sun (1996) presents a complete 1-dimensional elastic continuum model that accounts for tension, bending and torsion, and that also includes cable/fluid inertia effects. As Euler angles are used to model the transformation between the local and inertial reference frames, some configurations of the cable lead to a singular transformation. The torsional inertia is also neglected (to reduce the computational load), and as a result, problems such as that described by Hover (1997) cannot be modeled.

The objective of this paper is to gain a fundamental understanding of the loop formation process, including dynamic effects. Let us consider a cable positioned between 2 supports. The first stationary, and the second moving toward the first. (Both cantilevered—built-in—and simple supports are considered.) Depending on the type of supports, we expect different modes of loop formation as well as possible dynamic instabilities as the loop pops out.

We begin by introducing a cable model as a 1-dimensional dynamic elastic continuum capable of supporting tension, 2-axis bending and torsion. Euler parameters are used to describe the orientation of the cable section. The resulting equations of motion constitute a nonlinear initial-boundary-value problem that is solved numerically using separate finite differencing in time and in space.

CABLE MODEL

A cable model, similar to that used by Sun (1992, 1996) and Burgess (1993) is developed. The cable is considered to be a 1-dimensional dynamic elastic continuum capable of supporting tension, 2-axis bending and torsion. In essence, the cable is treated as a very slender (possibly nonuniform) elastica that is allowed to undergo arbitrarily large rotations. The cable is assumed to be homogeneous, isotropic, elastic and inextensible. The cross-section of the cable is circular. The external forces applied on the cable are weight, buoyancy, fluid drag and added fluid inertia.

To develop the equations of motion, 2 reference frames are used, as shown in Fig. 1. The inertial reference frame $(O, \vec{e}_1, \vec{e}_2, \vec{e}_3)$ is fixed in space and the natural choice for describing the cable acceleration components and weight. The local reference frame $(M, \vec{a}_1, \vec{a}_2, \vec{a}_3)$ is a local reference frame attached to the cable and the natural choice for describing the tension, torque, 2-axis bending, 2-axis shear and hydrodynamic forces. Here, $\vec{a}_1$ is the unit normal vector, $\vec{a}_2$ the unit binormal vector, $\vec{a}_3$ the unit tangent vector.

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