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Propagation of Flexural Waves at the Interface Between Floating Plates

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ABSTRACT

We present a theoretical study of wave propagation in 2 adjoining floating plates of different flexural rigidity, such as fast-ice sheets that abut across a pressure ridge, etc. Analytical formulas for the transmission and reflection coefficients for various conditions at the interface are obtained using the Wiener-Hopf technique for the case of semi-infinite elastic plates joined by a straight line. Solutions are scaled using the well-known characteristic length and time for an infinite floating plate, then categorized according to nondimensional wave frequency that is independent of physical parameters. Thus, the results give a relationship between scale-model experiments and real-sized structures.

INTRODUCTION

The shore-fast sea ice around Antarctica seasonally forms vast sheets that appear relatively featureless and homogeneous to the casual observer. However, closer inspection reveals that the ice sheets have substantial inhomogeneity at many length scales.

The ice sheet is typically a composite of floes that have frozen together after initial freezing and apparent breakup during winter. Each floe is typically tens of metres or kilometres across and often has very uniform thickness, giving an abrupt change of thickness between some adjacent floes. The abutting of floes can occur across regions of frozen seawater or ice rubble, or with pressure ridges when the floes are forced together, or partially refreezing cracks when the floes are moving apart. Stresses within the ice sheet, due to wind or current forcing, or due to thermal effects, cause further inhomogeneity in the form of pressure ridges, cracking or finger jointing. These structures are most visible around islands or headlands where stress concentration occurs. However, cracking in the ice sheet, perhaps only partially through the thickness of the sheet, is prevalent and can occur with tens of metres' spacing by the summer, when wave-induced breakup typically occurs. Brine drainage pores give further structure to the ice sheet over metre scales.

The flexural gravity waves that propagate in the ice sheet, corresponding to ocean waves, have wavelengths of tens to hundreds of metres for typical thicknesses of first-year sea ice. Consequently the wave propagation, in the ice sheet at least, occurs in a medium with significant structure over the scale of wavelengths. Our interest is in modelling flexural wave propagation in an inhomogeneous ice sheet with particular interest in the scattering of wave energy, and the effective mechanical properties of a homogeneous sheet that would allow calculation of the unscattered wave component. In this paper we take a first step towards a comprehensive model by solving for the change in wave field across a single straight-line interface between large floes, allowing for changes in thickness with an open (cracked) or joined (refrozen) interface.

Mathematical modeling of ice sheets of large scale in a marginal ice zone is mainly concerned with propagation of wave energy from the ocean, which affects the breaking-up process of the ice sheets. For many years, the formation and breakup of sea ice have interested not only geophysicists but also mathematicians (Balmforth and Craster, 1999; Evans and Davies, 1968; Fox and Squire, 1994; Gol'dshtein and Marchenko, 1989; Marchenko, 1997; Squire, Robinson, Langhorne and Haskell, 1988), because despite a great deal of idealization of the physical conditions, there have been few analytical solutions to the boundary value problems. Here we present examples of analytical solutions of 1 category of boundary value problems related to the dynamics of sea ice sheets, and from that solution we consider scaling effects on the interaction between 2 ice sheets.

In this paper we study the wave propagation on the surface of an ocean which is covered with 2 semi-infinite ice sheets or very large floating structures (VLFS) which give elastic responses for deflection of small amplitude and long wave length. The 2 plates are assumed to have different flexural rigidity and to be joined at an infinite straight line which we call transition or interface. Our main focus is on the reflection and transmission coefficients of plane waves when a plane wave of a given frequency is obliquely incident from infinity.

We use the Wiener-Hopf technique (Noble, 1958) to derive analytical solutions of the problem. Evans and Davies (1968) used the Wiener-Hopf technique (Noble, 1958) to derive analytical formulas of the reflection and transmission coefficients of waves propagating from a free ocean surface to a semi-infinite ice sheet. However, the formulas were explained to be too complicated for the numerical computation at the time. The first computation of formal solutions was shown by Fox and Squire (1990) using a computational mode matching technique. In Balmforth and Craster (1999), numerical computation of analytical solutions is carried out using quadrature computation of integral transforms. Chung and Fox (2002) showed that the solutions obtained by Evans and Davies (1968) could in fact be computed without numerical computation of integral transforms by finding roots of the dispersion equations. The formulas given in this paper can be directly implemented to computer codes without any numerical computational considerations such as numerical integrations.

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