

Wave Duration/Persistence Statistics, Recording Interval and Fractal Dimension

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ABSTRACT

The statistics of sea-state duration (persistence) have been found to be dependent upon the recording interval Δt . Such behavior can be explained as a consequence of the fact that the graph of a time series of an environmental parameter such as the significant wave height has an irregular, so-called fractal geometry. The mean duration, $\bar{\tau}$, can have a power-law dependence on Δt as $\Delta t \rightarrow 0$, with an exponent equal to the fractal dimension of the level sets of the time series graph. This recording interval dependence means that the mean duration is not a well-defined quantity to use for marine operational purposes. A more practical quantity may be the useful mean duration, $\bar{\tau}^u$, estimated from the formula $(\sum \tau_i^2)/(\sum \tau_i)$, where each interval $[t_i, t_i + \tau_i]$ satisfying the appropriate criterion is weighted by its duration. These results are illustrated using wave data from the Frigg gas field in the North Sea.

INTRODUCTION

The duration or persistence statistics of sea-state and other environmental parameters are important for purposes such as marine engineering operations, in which, for example, useful work can be performed only if the significant wave height h is less than a particular value h_0 . Over recent decades, several observational and theoretical studies have been undertaken in order to relate the statistical behavior of the duration of various sea-state criteria to, for example, the probability distribution of the wave height, its seasonal variation, and other parameters (e.g. Houmb and Vik, 1975; Graham, 1982; Mathiesen, 1994; Tsekos and Anastasiou, 1996; Soukissian and Theochari, 2001). In this paper we shall consider the collection of time intervals, in which $h(t) < h_0$: the intervals having durations τ_i , where $i = 1, 2, \dots, N$. The mean duration, $\bar{\tau}$, is calculated using the formula:

$$\bar{\tau} = (1/N) \sum_i \tau_i. \quad (1)$$

We can, of course, choose other criteria, for example, $h \geq h_1$, $h_0 \leq h < h_1$, etc.

If $h(t)$ is differentiable with respect to t , the mean duration is related to the probability distribution of h and dh/dt by the Rice-Kac formula (e.g. Rice, 1944; Kac, 1943; Mathiesen, 1994):

$$\bar{\tau}(h_0) = \frac{2F_h(h_0)}{f_h(h_0)E[|dh/dt| | h = h_0]}, \quad (2)$$

where F_h is the cumulative distribution function of h , i.e., $F_h(h_0)$ is the probability that $h \leq h_0$; $f_h(h_0) = dF_h(h_0)/dh_0$ is the probability density function of h ; and $E[|dh/dt| | h = h_0]$ is the expectation (mean value) of the absolute value of dh/dt , given that $h = h_0$.

However, time series of observed environmental parameters often have a noisy, irregular appearance, so that measurements

recorded at frequent intervals show considerable structure which does not appear if the measurements are recorded less frequently. In such a case, the time derivative is not well-defined, so the Rice-Kac formula of Eq. 2 cannot be applied. Although this may partly be due to the effect of errors in the measurement or of sampling variability, the phenomenon has been recognized as a manifestation of fractal behavior, shared with such phenomena as the irregularity of coastlines, the surfaces of snowflakes, and Brownian motion (Mandelbrot, 1983). It is a general property of such fractal objects or curves that their irregular shape remains even when you examine them at finer and finer scales. A fractal curve, such as a coastline, does not have a well-defined length, and if you measure it with rulers of finer and finer length, the total length will increase without bound. In order to measure such irregular objects, it is helpful to generalize the concept of dimension to noninteger values, and this was done by Hausdorff and Besicovitch in the early part of the 20th century (e.g. Hausdorff, 1919). A fractal curve, since it has an unbounded length but does not fill a plane, has a fractal dimension (Hausdorff-Besicovitch dimension, or just Hausdorff dimension) greater than 1 but less than 2; a fractal surface (e.g. a fracture surface) will have a fractal dimension between 2 and 3, and so on. Feder (1988) showed, by analyzing wave data from the Norwegian continental shelf, that a time series of significant wave height can have a fractal behavior. More details on the definition and calculation of fractal dimension are given below.

If the graph of an environmental parameter $h(t)$ has fractal behavior, it will have a fractal dimension between 1 and 2. If this graph crosses a horizontal straight line, it will intersect the line infinitely many times. If we consider a small interval around 1 of the intersection points, there will always be more intersection points within the interval, no matter how small the interval. The collection of points where $h(t)$ intersects the line $h = h_0$ is called the level set of $h(t)$ at h_0 , and it has a fractal dimension between 0 and 1. A point has dimension zero, as has a collection of points which are all more than a certain distance apart, but this particular collection of points will be so numerous and irregularly distributed that it will have a fractional dimension strictly greater than zero.

Since the number of intervals satisfying $h < h_0$ is infinite, any estimate of $\bar{\tau}$ using values of $h(t)$ sampled at successive recording

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