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Markov Theory for Groups of Very High Waves

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ABSTRACT

Statistics of wave groups among very high waves are examined using Markov theory and data from wave records developed by computer simulations of a Jonswap 3.3 sea. Very high waves are considered to be those with threshold heights exceeding the significant wave height by 10% or more. Theory indicates decreased group formation among the very high waves as a result of decreased Markov correlation between waves. Data confirm the prediction and provide additional insight into the group phenomena.

INTRODUCTION

Studies of wave groups in random seas generally involve consideration of the statistical description of runs of one or more high waves. Theory for such description is characterized by the Rice envelope theory of random noise or the Markov theory for statistically correlated events. Work based on the Rice theory has been described by Nolte and Hsu (1972), Ewing (1973), Goda (1976), and Longuet-Higgins (1984), among others; and work based on the Markov theory has been reported by Kimura (1980), Battjes and van Vledder (1984), Longuet-Higgins (1984), van Vledder (1992), Dawson et al. (1996), and Rodrigues et al. (2000), among others. Extension of the Rice theory to deal explicitly with group statistics, that is, statistics for runs of 2 or more high waves, has been described by Ochi and Sahinoglou (1989). A similar extension has been made by Dawson (1997) for the Markov theory. Experimental studies of these extensions have been reported by Dawson et al. (1991) and Dawson (1998a).

Detailed work on group phenomena has generally been limited in the past to high waves defined as those with threshold heights equal to the significant wave height of the random sea. The present paper describes additional work on group statistics of very high waves in random seas, that is, waves with threshold heights exceeding the significant wave height by 10% or more. The work utilizes Markov theory and data from wave records developed by computer simulations of a Jonswap 3.3 sea.

MARKOV THEORY FOR WAVE STATISTICS

Markov theory for runs of waves in random seas involves the basic assumption that statistical correlation exists only with immediately preceding waves. High waves in the theory are considered to be those with heights or crest amplitudes above a specified threshold level. The probability P_+ that a high wave is followed by a high wave and the probability P_- that a low wave is followed by a low wave are fundamental measures in the theory. The average number of waves in a run of one or more high waves, say, $\overline{N}_D$, and the average number of waves between beginnings of such runs, say, $\overline{N}_I$, are expressible in terms of these probabilities as:

$$\overline{N}_D = \frac{1}{1-P_+}, \qquad \overline{N}_I = \frac{1}{1-P_+} + \frac{1}{1-P_-} \tag{1}$$

Kimura (1980) has given theory for calculating the probabilities P_+ and P_- in terms of the correlation coefficient for wave heights associated with a specified sea. The calculation has since been generalized by Battjes and van Vledder (1984) and Longuet-Higgins (1984) so as to make it dependent on spectral properties of the sea rather than the correlation coefficient. In addition, it has been noted (Dawson et al., 1996) that the 2 averages $\overline{N}_D$ and $\overline{N}_I$ are not independent, but rather are related through the Rayleigh probability P, which gives the ratio of the number of high waves to total number of waves in a *representative* wave record, or, equivalently, the average ratio from a number of typical wave records. Thus, if n_o denotes the number of runs of one or more high waves in such a record, the number of high waves is expressible as $n_o\overline{N}_D$ and the total number of waves is expressible as $n_o\overline{N}_I$. The Rayleigh probability is then expressible as $P = \overline{N}_D/\overline{N}_I$, and the probabilities P_+ and P_- are found expressible from Eq. 1 as:

$$P_+ = 1 - C(1-P), \qquad P_- = 1 - CP \tag{2}$$

where C denotes the so-called Markov coefficient. A specialized version of these equations was first given by Longuet-Higgins (1984) using a different argument for derivation. On substitution of these relations into Eq. 1, the average number of waves in a run of high waves, and the average number of waves between beginnings of the runs are found expressible as:

$$\overline{N}_D = \frac{1}{C(1-P)}, \qquad \overline{N}_I = \frac{1}{CP(1-P)} \tag{3}$$

The Rayleigh probability in these equations measures the fraction of high waves that equal or exceed a specified threshold level in a representative wave record, and the Markov coefficient measures the statistical correlation existing for a given sea and specified threshold level. A value of unity for the Markov coefficient corresponds to no correlation, and lesser values correspond to increasing correlation. To the extent that these measures are left unspecified, Eqs. 2 and 3 represent exact relations based on the Markov theory. Approximations arise in specifying the form of relations for the Rayleigh probability and the Markov coefficient.

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