International Journal of Offshore and Polar Engineering
Vol. 11, No. 4, December 2001 (ISSN 1053-5381)
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Numerical Examination of Third-order Wave Force on Axisymmetric Bodies

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ABSTRACT

Within the frame of potential theory and the assumption of weak nonlinearity of wave motion, a numerical method is developed for the third-order triple-frequency wave loads on fixed axisymmetric bodies in monochromatic incident waves. Applying the numerical code, numerical computations were carried out for surge and heave forces and pitch moments on a uniform cylinder, truncated cylinders and a hemisphere. Examinations were made of the contribution to third-order forces and moments from potentials at each wave order, and the relation of third-order forces and moments to wave number and drafts of cylinders.

INTRODUCTION

It is believed that the third- or higher-order wave forces are the exciting sources for ringing responses of tension leg platforms (TLP) and gravity-based structures (GBS). Recently, a number of studies has been carried out for predicting third-order surge forces on cylinders. Based on the phenomenon that ringing occurs in long waves, Faltinsen, Newman and Vinje (1995) proposed a slender-cylinder theory. In such a long wave regime, the second-order diffraction can be neglected, and the third-order surge force is predicted by a kind of extension of the Morison equation. The attempt at full diffraction theory in the frequency domain was first made by Malenica and Molin (1995). They developed a semi-analytic solution for uniform cylinders in finite water depth, and successfully computed the third-order surge forces on them. Teng and Kato (1996) developed a numerical model for axisymmetric bodies by an integral equation method, which works for third-order surge forces, heave forces and pitch moments. Kim et al. (1998) developed a method for solving the full nonlinear wave diffraction from a uniform cylinder in the time domain. The third-order surge forces are then obtained by a Fourier analysis of the time history of full nonlinear wave forces. This method is also available for higher-order wave forces.

In this paper, the Teng and Kato method (1996) has been used to compute third- order triple-frequency surge force, heave force and pitch moment on cylinders and hemispheres. Examinations are made of the relation of the third-order forces and moments to body shapes, drafts of cylinders and wave frequencies, and the ratios of contributions to the third-order force and moments due to velocity potentials at each wave order. Numerical results show that:

- the third-order surge force on a truncated cylinder with a deep draft is close to that on the uniform cylinder with the same radius at the same water depth, and the third-order surge force on a hemisphere is smaller than that on the truncated cylinder with the same radius and draft;
- the third-order heave force on a hemisphere is larger than that on the truncated cylinder with the same radius and draft;
- the third-order pitch moment on the examined uniform cylinder has very big value at low frequency, and that on the truncated cylinder with a deep draft is close to that on the corresponding uniform cylinder at high frequency, but is very different at low frequency.

ANALYSIS METHOD

Free-Surface Condition

We consider the problem of wave diffraction from a vertical body of revolution, and define a right-handed coordinate system (x, y, z), with origin at the revolution axis of the body, $z = 0$ on the still, free surface and the z-axis pointing upward (Fig. 1). The fluid is assumed to be homogenous and incompressible, and the motion irrotational. There exists a velocity potential that satisfies the Laplace equation and the nonlinear free-surface boundary condition:

$$\Phi_{tt} + g\Phi_z + \frac{\partial}{\partial t}\,|\nabla\Phi|^2 + \frac{1}{2}\nabla\Phi\cdot\nabla|\nabla\Phi|^2 = 0 \tag{1}$$

on the free surface $z = z\,(x,y,t)$, defined by:

$$\zeta = -\frac{1}{g}\Phi_t - \frac{1}{2g}\left[\nabla\Phi\cdot\nabla\Phi\right]. \tag{2}$$

Under the assumption of weak nonlinearity, we can write the

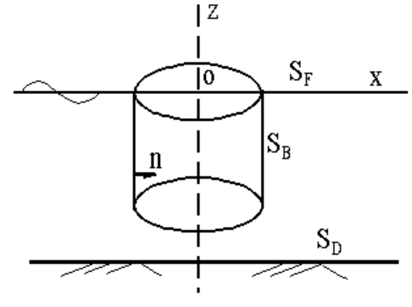

Fig. 1 Sketch of definition

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Received July 6, 2000; revised manuscript received by the editors October 30, 2001. The original version (prior to the final revised manuscript) was presented at the Tenth International Offshore and Polar Engineering Conference (ISOPE-2000), Seattle, USA, May 28–June 2, 2000.
KEY WORDS: Wave force, nonlinearity, axisymmetric bodies.