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Analysis of Ocean Waves by Crossing and Oscillation Intensities

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ABSTRACT

In this paper, relationships between wave characteristics and upcrossing and oscillation intensities are used to construct conservative bounds for significant wave heights and amplitudes and for moments of waves' crests and troughs. Results are illustrated by 5 examples in which both Gaussian and non-Gaussian models for a sea or a load are considered.

BASIC DEFINITIONS AND RESULTS

Let $x(t)$ be the height of the sea level at a fixed point as a function of time t. In oceanographic applications $x(t)$ is often seen as a sequence of waves where each wave can be described by means of its highest and lowest values (crest, trough), or by means of its height (= crest-trough) and wave period, describing the duration of a single wave. There is no general agreement about the formal definition of a wave. Often one uses the so-called mean downcrossing wave, where the wave is a part of a function between the consecutive downcrossings of the mean sea level:

Definition 1 *(Reference level $u_* = 0$) Assume $x(t)$ crosses a fixed reference level u_* finitely many times (here the level most frequently crossed by x). Denote by t_i, $0 < t_1 < t_2 < \ldots$, the times of downcrossings of u_*. The trough and crest m_i^*, M_i^*, say, of the ith wave are the global minimum and the global maximum of $x(t)$, $t_i < t < t_{i+1}$, respectively (Fig. 1). The so-called crest front amplitude H^* is defined as the difference between the crest and the trough, i.e.:*

$$H_i^* = M_i^* - m_i^* = \max_{t_i<t<t_{i+1}} x(t) - \min_{t_i<t<t_{i+1}} x(t) \tag{1}$$

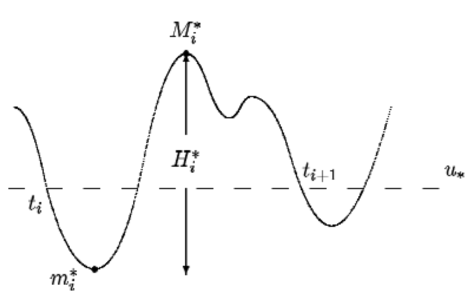

Fig. 1 Definition of mean downcrossing wave

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A simple measure of the severity of waves are the significant wave amplitude and the significant crest height, denoted by H^s, M^s, that is, averages of the highest one-third of amplitudes H_i^*, crest heights M_i^*, respectively. A more complete statistical characteristic of waves are (crest, trough) and (wave period, amplitude) distributions.

Two major uses of wave characteristic distributions are to predict extreme waves and fatigue lifetimes of marine structures. Clearly, using the mean crossing waves one neglects small oscillations superimposed on major waves. However, for fatigue accumulation in marine structures, it is well known that even small oscillations can contribute to the damage and hence must be analyzed. It is generally agreed that the fatigue is a rate-independent process, which means that only the sequence of turning points is essential for fatigue-life prediction and not the exact path of x between the local extremes. However, it takes at least 10^4 local maxima before fatigue failure, and often more than 10^7 . Consequently in practice it is not possible to use the whole structure of the sequence of local maxima and minima to predict the fatigue lifetime. Instead, the distribution of the so-called rainflow cycles, which are local maxima and minima of x, paired using the hysteresis properties of the material, is used (Brokate and Sprekels, 1996). The rainflow method was introduced by Endo, the first paper in English being by Matsuishi and Endo in 1967. Here the definition given in Rychlik (1987) will be used.

Definition 2 *To define the rainflow cycles, each local maximum in the load sequence is paired with one particular local minimum, determined as follows: From the ith local maximum (value M_i) one determines the lowest values in forward and backward directions between M_i and the nearest points at which the waveform exceeds M_i. The larger (less negative) of those 2 values, denoted by m_i^{rfc}, is the rainflow minimum paired with M_i, i.e. m_i^{rfc} is the least drop before reaching the value M_i again on either side. Thus the ith rainflow pair is (m_i^{rfc}, M_i) and the rainflow amplitude is $H_i^{rfc} = M_i - m_i^{rfc}$ (Fig. 2).*

As is evident, the rainflow minimum does not necessarily immediately follow or precede the corresponding maximum, so that the cycle (m_i^{rfc}, M_i) can include a number of oscillations. However, it may be shown that each minimum corresponds to only one maximum in forming a rainflow pair.

Other important characteristics of sea state (or a load process), which contains fundamental information about waves and cycles, are upcrossing and oscillation intensities, defined as follows.