

## Resistance in Unsteady Flow: Search for an In-line Force Model

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### ABSTRACT

**This paper offers an improvement to the Morison, O'Brien, Johnson and Schaaf (MOJS, 1950) representation of the in-line force for a sinusoidally oscillating flow. The modification proposed here consists of the addition of a third term, without the introduction of additional empirical coefficients. The 3-term model is expected to offer greater universality and higher engineering reliability, particularly in the inertia-drag regime where the original MOJS equation fails.**

### INTRODUCTION

Unsteady flows over bluff bodies arise in many engineering situations and the prediction of the fluid/structure interaction (forces and dynamic response) presents monumental mathematical, numerical and experimental challenges (Stokes, 1851; Wang, 1968; Sarpkaya, 1985, 1986a, 1992).

Stokes' classical solutions (1851) formed the basis of many subsequent models where the oscillations are presumed to be small enough to allow convective accelerations to be ignored. An extensive review (Mei, 1994) of the existing force models for flows at relatively small Reynolds numbers has shown that the degree of empiricism increases with increasing Reynolds number and some measure of motion unsteadiness.

In a sinusoidally oscillating flow, defined by  $U = U_m \sin 2\pi t/T$ , about a cylinder of diam  $D$ , every  $Re$  (Reynolds number =  $U_m D/\nu$ ),  $K$  (Keulegan-Carpenter number =  $U_m T/D = 2\pi A/D$ , and  $k/D$  (relative sand roughness) combination represents a unique flow situation. Here  $U_m$  is the maximum velocity in a cycle,  $T$  and  $A$  are the period and amplitude of oscillation, and  $\nu$  is the kinematic viscosity of fluid. As yet a theoretical analysis of the problem for separated flow is difficult and much of the desired information must be obtained experimentally and, if possible, numerically.

The computational representation of a turbulent motion, without proper physics, is a major roadblock to the ultimate promises of Computational Fluid Dynamics (CFD). Equally important is the fact that in highly complex, separated, time-dependent turbulent flows with large-scale unsteadiness about small bodies (with negligible diffraction effects), such as those considered here, the Reynolds number varies between zero and a desired maximum during a given cycle and the flow is 3D. Even if one were to develop rationally constructed, 2- or higher-order-equation turbulence models, there is no assurance that they will yield reliable results for time-dependent flows. There lies the difficulty of the numerical simulation of unsteady nonequilibrium turbulent flows, with or without separation. As it stands, the state of the art is less than satisfactory, not only because of the limitations of computational resources and lack of understanding of the physics of turbu-

lence, but also because the real-time control applications (remotely operated vehicles; multiple-link, multi-degree-of-freedom, underwater manipulators) demand hydrodynamic forces and torques in real time as a function of the state and state derivatives of the system.

The control of unsteady vehicle motions cannot, by their very nature, wait for day-long computer solutions or for the improvement of the current state of the computational art. To make matters worse, the motions of underwater manipulators are often short enough to occur in transient, rather than in quasi-steady states. Thus, the intelligent control of such vehicles requires the determination of the forces and torques predominantly during these transient states. This is a very demanding challenge to experimental and computational fluid dynamics communities and points up the necessity for parallel studies in the years to come, for the pursuit of demonstrably sound semi-empirical approaches (perhaps something better than the 1950 MOJS equation) for the prediction and execution of minimum-time or minimum-energy trajectories in complex environments. In this respect, the MOJS experimental studies on the forces on piles due to the action of progressive waves have provided a useful and somewhat heuristic first approximation.

### THE MOJS EQUATION

In a paper submitted to the Petroleum Transactions of AIME on 23 October 1949, Morison, O'Brien, Johnson and Schaaf (1950) wrote: "The force exerted by unbroken surface waves on a cylindrical object, such as a pile, which extends from the bottom upward above the wave crest, is made up of two components, namely: (1) A drag force proportional to the square of the velocity which may be represented by a drag coefficient having substantially the same value as for steady flow, and (2) A virtual mass force proportional to the horizontal component of the accelerative force exerted on the mass of water displaced by the pile. These relationships follow directly from wave theory and have been confirmed by measurements..." Thus was born the MOJS equation:

$$F = \frac{1}{2} \rho C_d D |U| U + \rho C_m \frac{\pi D^2}{4} \frac{dU}{dt} \quad (1)$$

where  $F$  is the force per unit length experienced by a cylinder of diameter  $D$ ,  $\rho$  is the density of fluid,  $U$  and  $dU/dt$  represent the undisturbed velocity and the acceleration of the fluid at the axis of

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