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How Big Are the Big Waves in a Gaussian Sea?

Krzysztof Podgórski
Department of Mathematical Sciences, IUPUI, Indianapolis, Indiana, USA

Igor Rychlik, Jesper Rydén and Eva Sjö
Centre for Mathematical Sciences, Lund University, Lund, Sweden

ABSTRACT

We discuss various aspects of statistical distributions for large waves. Spatial zero-level downcrossing waves evolving in time are considered. We call such a wave extremal if its crest height attains a local maximum in time. For extremal waves, the joint distribution of the wave length and the crest height is obtained. Generally, it is observed that taking into account time dynamics of spatial characteristics results in distributions different from those obtained for static records. Some other statistical issues for large waves are also discussed, including the Rayleigh model for the crest height, the evolution of wave groups, and the myth of the seventh wave.

NOTATION

$W(x,t)$	: sea elevation at position x and time t
$S(\omega,\theta)$	: directional spectrum of sea elevation $W(x,t)$ at an angular frequency $\omega > 0$ and an azimuth $\theta \in (-\pi,\pi)$
λ_{kl}	: the kth (in κ) and lth (in ω)spectral moment of S
κ	: wave number $\omega^2 \cos\theta/g$
$V(X)$	: variance of a random variable X
L, L_1, T	: spatial wave length, half-length, temporal wave period, respectively
S_1, H_1	: position and height of a crest (spatial or temporal)
N	: number of individual waves in a wave group
v_W, v_G	: individual wave and group velocity, respectively
M	: height of local maximum

INTRODUCTION

In marine folklore it is said that some waves are much higher than the others. It is sometimes referred to as the *myth of the seventh wave*, that is, that every seventh wave is higher than its 6 predecessors. The phenomenon has also been mentioned in fiction and on film as, for example, in Charrière (1970). Although the number 7 is a myth, some regularity of occurrence of high waves can provide a reason for this belief. For example, observing the sea surface one can see wave groups approaching the coast. This is a result of the dispersion of sea waves, which causes waves with similar frequencies to travel together, often far away from a region of storm. Individual waves move faster than the group, often dying out when leaving it while, when arriving from behind, they enter the group and grow,reaching their highest magnitude and giving the impression that the highest waves are coming in some regular pattern.

Unexpectedly large waves, *freak waves*, have been observed, causing damage to offshore platforms and structures. Such events have been observed with fixed position wave recorders. Robin and Olagnon (1991) examined nearly 11,000 eighteen-minute data records containing a total of 2 million waves and did not find any evidence of unexpectedly large wave heights in relation to heights predicted by the Rayleigh model. The conclusion can be that the frequency and height of large waves are in agreement with the intensity of crest heights modeled by the Rayleigh distribution. One could dispute and question if the measuring instruments have registered all extremal waves.

Truly, sea dynamics in vicinity of extremal waves as well as wave groupings observed close to the coast need nonlinear and hence non-Gaussian wave theory to be adequately modeled. However, in this paper we shall disregard the nonlinear effects and investigate a possibility of formal investigation of both myths through analysis of densities of wave characteristics for a simplified linear model, i.e. for a Gaussian sea. The myth of the seventh wave can be conceptualized by introducing the notion of a wave group. We demonstrate that the average number of waves in a group can be computed and, under suitable conditions, accidentally even be equal to 7.

From the formal standpoint, wave groups are difficult objects to study. Under special circumstances, when the sea surface is narrow-banded and long-crested, wave groups may be identified. For a confused sea there is a less noticeable organized movement of waves and thus each large extremal wave can be taken as a crest of a wave group. We show that appropriately defined extremal waves are not following the Rayleigh model, and indeed they can be essentially larger than predictions from the model. This does not stand in contradiction to the real-life records, as the "largest waves" are not registered in the frequencies suggested by the theoretical model if measurements are taken at a fixed location. The reason is that the sea elevation is registered by measuring devices such as a buoy or a distance radar only at a single point in space or in its direct vicinity. The distributions related to such measurements are different from those obtained for complete spatio-temporal records. The difference is well known to a surfer who is not simply waiting for a big wave, but rather constantly changes his position in a search for the "right spot" to meet a large wave. Similarly, a photograph of the sea profile along a line almost surely will not contain any waves that are at their extremal height. In order to register them one needs rather a movie showing the evo-

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