Strength and Deformation Capacity of Bends in Pipelines

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ABSTRACT

Bending moments on bends in pipelines may cause severe ovalization and stresses, both in longitudinal and circumferential directions. The bending stiffness depends not only on the radius of the bend, the diameter-to-wall-thickness ratio and the internal or external pressure, but also on the length of the bend, the loads due to the surrounding soil and the bending direction. After start of yielding, the bending moment may still increase considerably. For example, it can be shown that for bends with a radius-to-diameter ratio equal to 3, the maximum moment may be 2 or 3 times greater than the moment where first yielding started. Also the rotation capacity (the ability to follow imposed deformation) is many times greater than elastic theory can predict. In this paper, the structural behaviour of bends is analyzed. Improved analytical models for elastic design and new analytical models for plastic design are presented. The results of the analytical models are compared with test results and the results of finite element calculations.

NOMENCLATURE

- $a_2, a_3, \ldots$: Coefficients for in-plane bending of smooth bends
- $C_{\alpha}, C_p$: Function of $\lambda$ and of $\lambda$ and $\psi$ respectively
- $D_u$: External diameter
- $D$: Average diameter $AE: D = D_u - t$
- $D_{op}, D_p$: Function of $\lambda$, and of $\lambda$ and $\psi$, respectively
- $\Delta D_v$: Change in vertical diameter (in plane of bend)
- $\Delta D_h$: Change in horizontal diameter (perpendicular to plane of bend)
- $E$: Modulus of elasticity
- $El_{bo}$: Flexural stiffness of a bend
- $f_1, f_2, \ldots$: Coefficients in analysis of bends
- $G_{\alpha}, G_p$: Function of $\lambda$, and of $\lambda$ and $\psi$, respectively
- $K$: Curvature due to bending
- $k_{op}, k_{p}$: Flexural stiffness factor for pressure $P = 0$ and for $P$, respectively
- $k_{\alpha}, k_{\psi}$: Reduced value of $k_{op}, k_{p}$, respectively
- $M_m$: Maximum moment if longitudinal yielding is governing mechanism
- $M_{pbo}$: Maximum moment if occurrence of plastic hinges in circumferential direction is governing mechanism
- $M_o$: Moment which develops when, on applying $Q$, angular rotation is kept 0
- $M_p$: Plastic moment $\rightarrow M_p = 4r^2\sigma_f$
- $M_{e1}, M_{e2}$: Moment at which value of $m_e$ is first, respectively, attained anywhere in circumferential direction of wall of pipe
- $m_e$: Plate moment per unit width at end of elastic region $\rightarrow m_e = r^2\sigma g/6$
- $m_p$: Plastic plate moment per unit width $\rightarrow m_p = r^2\sigma g/4$
- $m_x, m_y$: Plate moment per unit width in longitudinal and circumferential directions, respectively
- $n_p$: Plastic plate normal force per unit width $\rightarrow n_p = r\sigma g$
- $n_x, n_y$: Plate normal force per unit width in longitudinal and circumferential directions, respectively
- $P$: Difference in pressure between inside and outside of pipeline $P = P_I - P_u$
- $Q$: Earth pressure
- $Q_d$: Directly transmitted earth pressure
- $Q_i$: Indirectly transmitted earth pressure (support reaction)
- $Q_{eq}$: Equivalent earth pressure to transform $Q_i$ to a quantity $Q_d$ which gives same average plate moments in circumferential direction as $Q_i$ does
- $R$: Radius of nonloaded bend
- $R'$: Radius of bend including additional curvature due to bending
- $R''$: Radius of bend including additional curvature and increased by $r$ for negative bending or decreased by $r$ for positive bending
- $r$: Average (or mean) radius $\rightarrow r = D/2$
- $t$: Pipe wall thickness
- $t_r, t_b$: Pipe wall thickness in straight pipe and bend, respectively
- $u, w$: Displacement of an element of pipe circumference along circumference and perpendicular to circumference, respectively; also $w = \Delta D/2$
- $\sigma K_{10l}$: Total increase in curvature in deformation step concerned
- $\sigma K_o$: Increase in curvature, due to ovalization
- $\sigma K_{\psi}$: Increase in curvature, due to longitudinal yielding
- $\sigma v$: Increase in ovalization, due to longitudinal yielding
- $\alpha, \beta, \gamma$: Loading angle, bearing angle for $Q_d$ and for $Q_i$ and $Q_{eq}$ respectively
- $\alpha$: Angle of unloaded bend
- $\Delta \alpha$: Change of $\alpha$
- $\eta$: Relative change of $\alpha$ to $\eta = \Delta \alpha/\alpha$
- $\eta_{e1}, \eta_{e2}$: $\eta$ associated with $M_{e1}$ and $M_{e2}$, respectively
- $\lambda$: Bend characteristic $\rightarrow \lambda = rR^2$
- $\nu$: Poisson’s ratio
- $\psi$: Bend characteristic $\rightarrow \psi = PR^2/Ert$
- $\psi_m$: Slope of yield surface

INTRODUCTION

Pipelines, offshore and onshore, are subjected to combinations of various loadings, such as internal or external pressure, surrounding soil, bending, normal force, shear force and sometimes...