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Effect of Additive Noise on Parametrically Excited Ship Rolling

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ABSTRACT

The "hopping" phenomenon in the phase space of a ship rolling model with two coexisting attractors, representing the heave and heave/roll responses, is investigated. The noise-induced erosion of basins of attraction is studied using the simple cell-to-cell mapping technique. The power spectra of a function, which monitors the hopping phenomenon and determines the phase durations of roll motions, are shown to exhibit 1/f noise, or power-law behaviour. The behaviour is interpreted using a recently proposed general principle based on the extremal dynamics; the related implication is also discussed.

INTRODUCTION

In recent years, attempts have been made to interpret the qualitative behaviour of ship instability in terms of the safe basin of attraction in the phase space (Thompson, Rainey and Soliman, 1990; Nayfeh and Sanchez, 1990). For the well-known instability phenomenon of large rolling motions of ships in longitudinal waves, the problem has been studied using coupled heave-roll models of parametric excitations (Sanchez and Nayfeh, 1990; Liaw, Bishop and Thompson, 1993). In the case when the natural frequency of a ship in heave is about twice its roll frequency and is close to the wave frequency, large-amplitude roll motions, attributable to "principal parametric resonance," and capsizing can occur even if there is no direct roll excitation.

The equations of motion governing the parametric resonance phenomenon described above are usually derived based on an idealized two-dimensional model of the ship. Heave excitations are necessarily simplified in order to keep the numerical efforts of solving the nonlinear dynamic problem to a manageable level. The heave excitations due to regular longitudinal waves are very often represented by harmonic forces. If only the instability conditions for parametric resonance are of interest, the roll angle responses are assumed to be small and the governing equations can then be simplified to either a Mathieu or Hill equation (Liaw, 1994). Analytical solutions for the instability conditions can be readily derived from the Mathieu or Hill equation.

However, if one is interested in the large roll responses and not just the conditions for roll responses to occur, nonlinear equations including large-angle effects have to be considered. Numerical analyses of the coupled heave-large roll system have shown that the system can have more than one steady-state response or attractor (Liaw et al., 1993) near the principal resonance region. Besides the heave-only response (H-attractor), which is the steady-state heave response without roll, there are also the coupled heave and subharmonic roll response of order 2 (HR-attractor). Each attractor has its own basin of attraction, i.e., the set of initial conditions which lead to the particular attractor. Within the resonance region, the ship can have large rolling motions, even without direct roll excitation, if its initial conditions are inside the basin of the HR-attractor.

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Following an idea suggested by Rainey and Thompson (1991), the stability behaviour of a ship can be evaluated by studying the evolution of its safe basin (e.g., the basin of the H-attractor). As pointed out by Sanchez and Nayfeh (1990), attractors associated with large roll responses can, in some cases, go through bifurcations and crises and lead to capsizing. The above-mentioned studies were all based on the assumption that the heave excitation is harmonic, or at least periodic. In actual sea environments, the heave excitation will not be periodic; noise is always present. It would therefore be interesting to know how the parametrically excited system is affected by additive noise in the heave excitation.

A significant number of research works have been presented in the last decade in the challenging area of noisy nonlinear dynamic systems, e.g., Moss and McClintock, 1989; Pike and Lugiato, 1987; Kapitaniak, 1988. For dynamical systems with multi-attractors, noise-induced hopping phenomena (Arecchi and Califano, 1984) are directly relevant to the problem of parametric resonance. This paper investigates the hopping behaviour in parametrically excited ship rolling when the system has two coexisting attractors, e.g., H and HR.

PARAMETRICALLY EXCITED SHIP ROLLING

As described in a previous paper (Liaw, 1994), the coupled heave and roll motions can be represented by the following equations of motion:

$$\ddot{\theta}+2\xi_r\omega_r\dot{\theta}+\omega_r^2\sin\theta\left\{1+\frac{1}{2GM_o}\left[\frac{1}{12}\left(\frac{b}{d}\right)^2\tan\theta^2+\left(\frac{z_f-z}{\cos\theta}\right)^2\right]\right\}=0 \tag{1}$$

$$\ddot{z}+2\xi_h\omega_h\dot{z}+\omega_h^2 z=F_o\sin\omega_o t \tag{2}$$

where the dots denote derivatives with respect to time t; θ and z are respectively the roll and normalized relative heave displacements; ω_r and ω_h , the linear roll and heave frequencies; ξ_r and ξ_h, the linear roll and heave damping ratios; d, b and z_f represent respectively the draught, beam and water surface elevation. The static metacentric height, GM_o, for a ship with a rectangular cross-section is given as:

$$GM_o=z_f-0.5+\frac{1}{12}\left(\frac{b}{d}\right)^2 \tag{3}$$

The governing equation for roll, Eq. 1, is coupled with the