

Application of a Higher Order BEM in the Calculation of Wave Run-Up on Bodies in a Weak Current

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ABSTRACT

The paper presents the calculation of wave run-up on a floating body in current by a higher order boundary element method. The method is based on the perturbation expansion of velocity potential and Green's function in terms of current velocity. Novel integral equations are employed, which improve the calculation of some of the Cauchy principal value integrations in the integral. In order to save computer time and storage, the numerical scheme has been designed to accept two, one or no planes of symmetry of the body geometry.

INTRODUCTION

Many researchers have developed efficient algorithms to evaluate the action on structures by waves and current individually. However, it is a common phenomenon in nature that waves and current coexist, or bodies move in waves. When a current and waves coexist, the free surface boundary condition will be changed. Accordingly, the diffraction and the radiation of waves from a body will be changed, and wave forces and wave run-up on structures will also be modified. The wave run-up is a dominant factor in the determination of the deck elevation of an offshore platform. An underestimated elevation will not assure the safe normal operation of the platform, and an overestimated elevation will increase the cost and decrease the stability of the platform.

In the calculation of wave diffraction and radiation around bodies, the integral equation method is widely used. For the wave and current problem, by using a Green's function (Wehausen and Laitone, 1960) which satisfies the free water surface and far field conditions, the integration domain can be limited to the body surface and a small area on the free water surface. Unfortunately, the calculation of the Green's function is time consuming, so efficiency of this method is greatly reduced as compared with the case without a current. In ship hydrodynamics, ships are assumed to be very slender and the strip theory is introduced. In essence, the strip theory converts a three-dimensional body into two-dimensional sections which are hydrodynamically independent. Numerical calculation of the flow about these independent sections is much simpler. Structures such as tension-leg platforms (TLPs), however, are bluff, so the slender assumption can no longer be applied. On the other hand, the forward speeds of these structures or current velocities are usually small relative to the incident wave celerity. For such a case, the perturbation method is widely used, in which the integral equation and the Green's func-

tion are expanded in terms of the current velocity (Zhao, Faltinsen, Krokstad and Aanesland, 1988; Huijsmans and Hermans, 1989; Grue and Palm, 1991; Eatock Taylor and Teng, 1993). By means of the expansion, the wave-current problem can be divided into two parts: (i) the case of waves alone and (ii) the correction due to the current. For the first problem, successful higher order boundary element methods have been developed (Chau and Eatock Taylor, 1988; Liu, Kim and Lu, 1991) based on the efficient calculation of the Green's function in still water (Noblesse, 1982; Newman, 1984, 1985). For the second problem, the first order term of the Green's function and its derivatives can be derived by the zero speed Green's function and its derivatives.

The use of the higher order panel method is thought to have certain benefit in the resulting panel computations. The body surface may be discretized into a set of curved panels, and the velocity potentials at the nodes become the unknowns. The velocity potentials and their derivatives inside a panel can be represented by nodal values and shape functions. This is advantageous for the calculation of wave run-up and second order forces on structures, where the velocity potential at the free surface and its derivatives are needed. When *constant* panels are used, these values may only be obtained by extrapolation from the panel centroids.

BOUNDARY VALUE PROBLEM

Let us consider the problem in a right-handed frame of reference translating with a small steady forward speed in deep water, or a weak current directed along the negative x-axis. z is measured vertically upwards and $z = 0$ is on the still water surface.

The fluid is assumed to be homogenous and incompressible, and the motion irrotational. The waves are assumed to be periodic. There exists a velocity potential Φ that satisfies the Laplace equation:

$$\nabla^2 \Phi = 0 \quad (1)$$

and

$$\Phi_{,tt} + 2\nabla\Phi \cdot \nabla\Phi_t + \frac{1}{2}\nabla\Phi \cdot \nabla(\nabla\Phi \cdot \nabla\Phi) + g\Phi_z = 0 \quad (2)$$

on the free surface $z = \eta$, which is unknown; and

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KEY WORDS: Wave run-up, wave and current, boundary element method.