International Journal of Offshore and Polar Engineering
Vol. 5, No. 3, September 1995 (ISSN 1053-5381)

The Response of a Thick Flexible Raft to Ocean Waves

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ABSTRACT

The solution to the problem of a thick flexible raft subjected to a train of incoming ocean surface waves is presented. It is found that the inclusion of transverse shear and rotary inertia has significant effects on the reflection and transmission coefficients of the body when its length-to-thickness aspect ratio is small and, predictably, that at higher aspect ratios a thin plate analysis is perfectly adequate. A major contrast between the thin and thick plate models is that the asymptotic limit of the raft response in the case of infinite stiffness is different; this leads to altered behaviour when the aspect ratio is small.

INTRODUCTION

Although the motions of rigid bodies in ocean waves have been studied in some detail (see Lee, 1976, for a typical analysis), this is not the case when the body suffers significant bending due to the underlying wave profile. In a series of recent papers Meylan and Squire (1993, 1994a, b) have presented results concerning the rigid body oscillations and bending of a thin, flexible, two-dimensional raft brought into motion by the action of ocean surface waves. They find that reflection and transmission coefficients are greatly affected by the geometry of the raft, with the reflection coefficient becoming zero in certain configurations, thereby leading to perfect transmission of the wave train past the body. At these *resonances* large strains are induced in the floating raft, as the coupling between it and the wave profile beneath is optimal. When the body is long in comparison to the wave's length, the separation of the resonances becomes regular and equal to half the wavelength of the flexural wave in the material. This situation is analogous to the reflection of electromagnetic radiation at a conducting interface. At shorter ratios, the separation is more complex.

While the thin plate model may be a reasonable analogue to a floating body in many instances, in some circumstances the effect of the body's thickness on the flexural behaviour cannot be neglected. This is especially true for rafts of small length-thickness ratio — hereafter called aspect ratio — where the effect of shear and inertia may become significant. In the present work we continue the theme of the Meylan and Squire papers but introduce a modified surface boundary condition to represent the floating raft, namely that corresponding to a thick plate which includes the effects of transverse shear and rotary inertia. This necessitates a change to both the plate equation and to the transition conditions defined at the two ends of the raft. The specific surface boundary condition used is based on the work of Timoshenko (1921) and Mindlin (1951), whose equations are derived via the three-dimensional equations of linear elasticity theory.

The current paper begins by presenting the nondimensionalized boundary value problem to be solved, including a full thick plate boundary condition defined over the raft's nondimensionalized length $0 \leq x \leq 1$ and open water boundary conditions outside this region. The problem addressed is two-dimensional and so considers only cylindrical plate bending (or the bending of a beam). A Green's function approach is then described which, via the Green's function for open water and Green's theorem, allows an integral equation to be derived which is solved numerically. Results are presented demonstrating the differences between the thin and thick plate renderings of the wave-induced motions.

THICK PLATE MODEL

The derivation in this section draws on the work of Meylan and Squire (1993, 1994a, b) which describes the motion of an ice floe under wave action, and is based on a thin elastic plate model for ice deformation. In that model the effect of rotary inertia and transverse shear is neglected as the finite thickness of the plate is ignored. The related paper of Fox and Squire (1991) is also of significance.

An equation relating the transverse displacement w and the transverse shear ψ of a thick plate to the applied normal pressure p has been derived by Mindlin (1951) from the three-dimensional equations of linear elasticity. The equation includes the effect of rotary inertia and transverse shear, and invokes plausible assumptions such as the plate faces remaining parallel. We shall consider a restriction of the Mindlin equation of motion to two dimensions, where it then becomes analogous to the beam equations of Timoshenko (1921), except for a change in the value of the flexural rigidity. The coupled equations for the shear ψ and displacement w are:

$$\left.\begin{aligned} L\frac{\partial^2\psi}{\partial x^2}-\mu^2Gh\left(\psi+\frac{\partial w}{\partial x}\right)&=\frac{\rho h^3}{12}\frac{\partial^2\psi}{\partial t^2} \\ -\mu^2Gh\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial\psi}{\partial x}\right)+\rho h\frac{\partial^2 w}{\partial t^2}&=p \end{aligned}\right\} \tag{1}$$

where ρ' is the density of the plate, h is its thickness and, for a shear modulus G and a Poisson's ratio ν, the flexural rigidity is $L = Gh^3/6(1 - \nu)$. The choice of the value of μ is in some sense arbitrary and Mindlin suggests two options. Either μ is chosen so that in the limit of high frequency the wave speed tends to the Rayleigh surface wave, or it is chosen such that the thickness-shear motion has the same frequency as predicted from the three-dimensional equations. The later value for μ^2 is $\pi^2/12$, which is the value we shall use. The boundary conditions for a free plate are:

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Received September 12, 1994: revised manuscript received by the editors July 14, 1995. The original version (prior to the final revised manuscript) was submitted directly to the Journal.
KEY WORDS: Thick plate, flexible body, ocean waves.