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Stability Studies on Cellular-Walled Circular Cylindrical Shells, Part I — Theoretical Analysis

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ABSTRACT

The buckling load capacity of a form of cellular-walled circular cylindrical shell suggested by fossil shell remains is analyzed. Shells under combined loading of axial compression and external pressure are studied. In particular, the effect of high fluid pressure within the cells on the buckling behaviour of the shell is considered. This study demonstrates that shells of this type with pressurized cells exhibit significantly improved stability and thus appear to have potential in engineering applications, particularly in marine situations.

NOMENCLATURE

A,B,C	: Amplitudes of a set of displacement functions u,v,w
a_i,b_i,c_i,d_i,f_i	: Coefficients in Eq. 1
c_1, c_2, c_3, c_4	: Coefficients in Eq. 15
c^2, δ	: Defined in Eq. 10
D_1, D_2	: Bending stiffness
E	: Young's modulus of isotropic material
E_1, E_2, G	: Extensional Young's moduli and shear modulus
Eb_1, Eb_2	: Bending Young's moduli
K_1, K_2	: Extensional stiffness
k, k_1	: Defined in Eq. 10
L	: Length of cylinder
$M_\alpha, M_\beta, M_{\alpha\beta}, M_{\beta\alpha}$	: Moment resultants
$2m,n$	: Number of half waves along circumferential and axial direction
$N_\alpha, N_\beta, S_\alpha, S_\beta$	: Stress resultants
P, p, p_1	: Axial compressive load, external pressure, cell pressure
P', p'	: Tensional loads due to cell pressure
P_{cr}, p_{cr}	: Critical buckling loads
Q_α, Q_β	: Transverse shear stress resultants
q_1, q_2	: Nondimensional loads
R_x, R_p	: $R_x = P/P_{cr}$, $R_p = p/p_{cr}$
R,t	: Radius of cylinder
r	: Radius of cells
S	: Cells spacing
t	: Thickness of cylinder
u,v,w	: Midsurface displacements
x,y,z	: Axial, circumferential and radial coordinates of cylinder
α,β,γ	: Dimensionless coordinates
$\varepsilon_\alpha, \varepsilon_\beta, \varepsilon_{\alpha\beta}$	: Normal strains and shear strain
$\eta_\alpha, \eta_\beta, \eta_{\alpha\beta}$	: Bending strains and twisting strain
λ	: $= \dfrac{n\pi R}{L}$
μ, μ_1, μ_2	: Poisson's ratios
$\sigma_\alpha, \sigma_\beta, \tau_{\alpha\beta}$	: Normal stresses and shear stress

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KEY WORDS: Circular cylindrical shell, fossil shell, stability, theory.

INTRODUCTION

This work was triggered by reports of fossil shell remains belonging to the Nautiloid Cephalopod group. These extinct animals (relatives of the modern pearly nautilus) grew to about 300 mm in length and had an exoskeleton in the form of a conical shell with a small apex angle. The unique feature of this shellfish was that it contained small and closely spaced holes running longitudinally in the shell wall (Banks, 1988). Paleontologists could provide no satisfactory explanation for the shellfish to have evolved in this form. Other than the obvious conclusion that a cellular wall has better bending stiffness than a solid wall of the same mass, there appeared to be no particular reason for the specific form of this shell. From a strength point of view, longitudinal holes would not appear to provide any particular advantage over circumferential or spiral holes, and spiral holes would be easier for the shellfish to manufacture.

Fig. 1 shows an artist's impression of the living shellfish together with a sketch of a section through the shell wall: this sketch is taken from actual fossil specimens. The structure is essentially a thin-walled circular shell with small, closely spaced holes running longitudinally through the wall. In addition, there is a large number of considerably smaller holes connecting these longitudinal channels with the outside of the shell. The purpose of the longitudinal holes and the smaller connecting holes is not clear. One likely explanation is that the longitudinal holes were filled with muscle while the very small connecting holes may have been for some form of nerve tissue (ie., sensing element). If it was the case, then, by contracting the muscle the shellfish could apply pressure to the inside of the holes. Large pressures could be achieved with muscle contraction, which, from a shell stability viewpoint, act in a similar fashion to a small internal pressure.

It is well-known that application of pressure to the inside of a thin, solid-walled cylindrical shell can increase its axial buckling load (Fung and Sechler, 1957; Harris, Suer and Skene, 1957; Lo, Crate and Schwartz, 1950). One contributing factor could be that the pressure tends to make the shell more nearly circular and hence reduces the size of geometric defects. Internal pressure obviously must also increase the external buckling pressure. Since it was expected that high cell pressure would act in a similar