International Journal of Offshore and Polar Engineering
Vol. 4, No. 3, September 1994 (ISSN 1053-5381)

Diffraction of Surface Waves by Axisymmetric Obstacles in Water of Finite Depth

Vitaly V. Yakovlev and Alexander V. Pyatetsky

Laboratory of Shelf Hydrodynamics, Institute of Hydromechanics, Ukrainian Academy of Sciences, Kiev, Ukraine

ABSTRACT

An approximate model is proposed for water wave transformation over bottom inhomogeneities in water of the finite variable depth. This model formed the basis for an effective numerical analytical method to calculate wave fields. Water wave interactions with large isolated axisymmetric obstacles or a set of the same ones set out on permeable and nonpermeable grounds are investigated. Comparisons have been made with available solutions relating to various specific configurations and good agreement has been obtained in all cases. The effect of wave energy focusing has been found, causing the formation of a bottom erosion zone in the shadow domain near isolated obstacles and the essential increase of wave loads on the shadowed body when the set of obstacles is considered.

INTRODUCTION

The investigation of wave action on complex-shape constructive elements is one of the most important problems in offshore design. The most commonly used way to calculate the wave loads on large obstacles is to apply the numerical (FEM or BEM) methods, which require essential CPU time and storage (Garrison, 1978; Zienkiewicz et al., 1978). However, carrying out wide calculations to choose the optimal shapes and arrangements of structure elements demands the development of appropriate approximate models, which are free from the above limitations.

In this paper is proposed the approximate quasi-three-dimensional linear model of wave transformation in water of finite variable depth. An effective numerical-analytical method is developed to calculate the wave fields and wave diffraction interactions with an isolated axisymmetric obstacle and a set of the same obstacles set out on permeable or nonpermeable grounds.

APPROXIMATE QUASI-THREE-DIMENSIONAL MODEL OF WAVE TRANSFORMATION IN WATER OF FINITE VARIABLE DEPTH

Consider the wave motions of inviscid incompressible fluid in a domain of variable depth $H_1(x,y)$ in a case of steady harmonic oscillations with the frequency ω and the amplitude A. The velocity potential $\Phi_1(x,y,z)$ satisfies the Laplace equation:

$$\nabla^2\Phi_1=0 \tag{1}$$

subject to the usual boundary conditions on the free surface and seabed:

$$\frac{\partial\Phi_1}{\partial z}-\frac{\omega^2}{g}\Phi_1=0, \qquad z=0 \tag{2}$$

$$\frac{\partial\Phi_1}{\partial z}+\nabla\Phi_1\cdot\nabla H_1=0, \qquad z=-H_1 \tag{3}$$

Introduce the dimensionless variables:

$$(x^*,y^*,z^*,H_1^*,A^*)=(x,y,z,H_1,A)/\lambda_0, \quad \Phi^*_1=\Phi_1\omega/(\lambda_0 g)$$

where $\lambda_0=g/\omega^2$ is the characteristic wave length for deep water. The stars are dropped for simplicity of notation.

In order to obtain the equation of wave transformation in water of finite variable depth, we use the Galerkin procedure, which allows the exclusion of the vertical "nonwave" coordinate z (Pelinovsky et al., 1984). For this purpose we chose an infinite sequence of the coordinate functions:

$$Z_i(x,y,z)=\cosh k_i(z+H_1)/\cos k_i H_1, \quad i=1,2,\ldots, \tag{4}$$

where $k_i(x,y)$ are the roots of the nondimensional dispersive equation $k\tanh kH_1=1$.

Moreover, consider the function $\Phi_1^{(N)}(x,y,z)$, which approximates the unknown velocity potential $\Phi_1(x,y,z)$ in a domain under consideration as:

$$\Phi_1^{(N)}(x,y,z)=\sum_{i=1}^{N}\phi_i(x,y)Z_i(x,y,z) \tag{5}$$

Following the Galerkin procedure, the coefficients $\phi_i(x,y)$ are determined from the requirement that the left part of Eq. 1 after substitution of $\Phi_1^{(N)}$ instead of Φ_1 must be orthogonal to the functions Z_i:

$$<Z_j,\sum_{i=1}^{N}\nabla^2(\phi_i Z_i)>=0 \tag{6}$$

After introduction of the scalar product:

$$<f,h>=\int_{-H_1}^{0}\kappa\, fhdz \tag{7}$$

with the weight function:

$$\kappa(x,y,z)=\frac{\cosh k_1H_1}{\cosh k_1(z+H_1)}$$

we obtain from Eqs. 1~3 and 6 for $N=1$ the equation (Yakovlev, 1985):

$$k_1^{-2}\nabla^2\phi_1+\left[2\nabla k_1^{-2}-(\cosh k_1H_1)^{-1}\cdot\nabla H_1\right]\nabla\phi_1+$$
$$\left[1+\nabla^2k_1^{-2}-\nabla(\cosh k_1H_1)^{-1}\cdot\nabla h_1-(\cosh k_1H_1)^{-1}\nabla^2H_1\right]\phi_1=0 \tag{8}$$

Received September 1, 1993: revised manuscript received by the editors January 10, 1994 . The original version (prior to the final revised manuscript) was submitted directly to the Journal.
KEY WORDS: Water wave diffraction, finite depth, axisymmetric obstacles, wave load, permeable seabed, bottom wave velocities.