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A Model for the Motion and Bending of an Ice Floe in Ocean Waves

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ABSTRACT

A new model to represent the wave-forced motion and flexure of a single elastic ice floe of constant thickness is reported. The model predicts that resonance, i.e., perfect transmission, occurs when the ratio of the ice wavelength to the floe diameter assumes certain values, analogous to the propagation of electromagnetic radiation through a homogeneous slab. Features in the reflexion and transmission coefficients, and strain fields are discussed.

FORMULATION OF PROBLEM

Our approach is based on Tayler (1986). A solitary ice floe of width L is modelled as an elastic raft floating on the equilibrium surface ($z = 0$) of the ocean, which is assumed to be infinitely deep. We consider the two-dimensional motion and flexure of the raft when acted upon by a train of small amplitude surface waves propagating in the direction of the positive x-axis. The raft occupies the region $0 \le x \le L$, $z = 0$.

Over the raft we use the Bernoulli-Euler model for the deflexion, neglecting gravity effects:

$$\frac{\partial^2 w}{\partial t^2}+\mu^2\frac{\partial^4 w}{\partial x^4}=\frac{P}{\rho' h} \qquad z=0,\ 0<x<L$$

where ρ' is the density of the raft, h is its thickness, $\mu^2 = Eh^2/12\rho'(1-v^2)$, where E is the effective Young's modulus and v is Poisson's ratio, w is the surface displacement of the water, P is the pressure on the water surface and g is the acceleration due to gravity. The bending moment and shear must vanish at the ends of the raft, i.e.:

$$\frac{\partial^2 w}{\partial x^2}=\frac{\partial^3 w}{\partial x^3}=0, \qquad \text{at } x=0 \text{ and } x=L \tag{1}$$

We nondimensionalize, scaling lengths by L and time by $\sqrt{L/g}$, and derive the following nondimensional boundary value problem, assuming the velocity potential ϕ is periodic in time with radian frequency $\sqrt{\alpha}$:

$$\left.\begin{aligned}
&\nabla^2\phi=0,\ \infty<z<0,\ \frac{\partial\phi}{\partial z}=0,\ z\to\infty\\
&\frac{\partial\phi}{\partial z}+\alpha\phi=0,\ z=0,\ -\infty<x<0,\ 1<x<\infty\\
&\frac{\partial\phi}{\partial z}+\alpha\phi=\alpha\gamma\frac{\partial\phi}{\partial z}-\beta\frac{\partial^5\phi}{\partial x^4\partial z},\ z=0,\ 0<x<1\\
&\frac{\partial^3\phi}{\partial x^2\partial z}=\frac{\partial^4\phi}{\partial x^3\partial z}=0,\ \text{at } x=0 \text{ and } x=1
\end{aligned}\right\} \tag{2}$$

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where

$$\gamma=\frac{\rho' h}{\rho L} \quad \text{and} \quad \beta=\frac{\rho' h\mu^2}{g\rho L^4}$$

together with conditions at $\pm\infty$:

$$\lim_{x\to\infty}\phi=Te^{-i\alpha x-\alpha z} \quad \text{and} \quad \lim_{x\to-\infty}\phi=e^{-i\alpha x-\alpha z}+\mathrm{R}e^{i\alpha x-\alpha z} \tag{3}$$

The third equation of Eq. 2, representing the boundary condition across the raft, is then rewritten as follows using a Green's function $g(\xi,x)$:

$$\phi_z(x,0)=-\frac{\alpha}{\beta}\int_0^1 g(\xi,x)\phi(\xi,0)d\xi, \qquad z=0,\ 0<x<1 \tag{4}$$

METHOD OF SOLUTION

A Green's function $G(\xi,\eta;x,z)$ for the half space which satisfies the open water boundary conditions is:

$$G(\xi,\eta;x,z)=\frac{1}{4\pi}\ln\left\{(\xi-x)^2+(\eta-z)^2\right\}-\frac{1}{4\pi}\ln\left\{(\xi-x)^2+(\eta+z)^2\right\}$$
$$-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{|\omega|-\alpha}e^{-|\omega|(\eta+z)}e^{i\omega(\xi-x)}d\omega \tag{5}$$

By means of Green's theorem in the plane we obtain:

$$\phi(x,z)=\frac{R}{2}e^{i\alpha x-\alpha z}+\left(\frac{1+T}{2}\right)e^{-i\alpha x-\alpha z}$$
$$+\int_0^1 G(\xi,0;x,z)\left\{\alpha\phi(\xi,0)-\phi_\eta(\xi,0)\right\}d\xi \tag{6}$$

Finally, substituting for the asymptotic limits of the potential and the Green's function (Eq. 4) under the raft, we obtain the following Fredholm integral equation for $\phi(x,0)$:

$$\phi(x,0)=\frac{i\alpha}{2}A(-\alpha)e^{i\alpha x}+\left(1+\frac{i\alpha}{2}A(\alpha)\right)e^{-i\alpha x}-\frac{\alpha}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-i\omega x}}{|\omega|-\alpha}A(\omega)d\omega \tag{7}$$