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A Hybrid Axial-Torsional Finite Element for Flexible Risers

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ABSTRACT

In this paper, the development of a finite element suitable for the three-dimensional, dynamic analysis of flexible marine risers is summarized. The conventional version of the element is described along with hybrid axial and torsional modifications. Results illustrating the performance of the element are presented for a single-catenary riser and a deep-water buoyant riser.

INTRODUCTION

The complexity inherent in the analysis of marine risers requires the use of reliable computational tools. Several analytical procedures pertinent to riser analysis have been developed during the last 15 years. A review of the literature to date is given by Boubenider (1992).

The present paper gives a partial account of a study undertaken at the Offshore Technology Research Center towards a finite element technique for the three-dimensional, dynamic analysis of rigid and flexible deep-water risers. Outlined below are 3 versions of a tubular beam finite element capable of modelling risers undergoing arbitrarily large displacements and rotations. Apart from the conventional formulation in which only the geometry of the element is interpolated (in terms of nodal position and orientation vectors), 2 hybrid modifications are summarized: One in which the axial force is interpolated, and another involving interpolation of both the axial force and the torsional moment. Two examples of application, a single-catenary riser and a deep-water buoyant riser, are discussed.

FINITE ELEMENT

The geometry of the 3-node, isoparametric, tubular finite element (see Figs. 1-2) developed in the course of the present study can be expressed in terms of (convected) material coordinates as:

$$\mathbf{x}=\sum_{k=1}^{3} N^{(k)}(\xi^1)\left\{\mathbf{x}^{(k)}+r^{(k)}(\xi^3)\left[\mathbf{V}_2^{(k)}\cos(\xi^2)+\mathbf{V}_3^{(k)}\sin(\xi^2)\right]\right\} \tag{1}$$

where ξ^1, ξ^2, ξ^3 are the longitudinal ($-1 \le \xi^1 \le 1$), circumferential ($0 \le \xi^2 \le 2\pi$) and radial ($-1 \le \xi^3 \le 1$) material coordinates; $\mathbf{x}^{(k)}$, $k = 1,2,3$ are the nodal position vectors; ($\mathbf{V}_1^{(k)}$, $\mathbf{V}_2^{(k)}$, $\mathbf{V}_3^{(k)}$), $k = 1,2,3$ are nodal (cross-section) orientation vectors (triplets of mutually orthogonal unit vectors: $\mathbf{V}_2^{(k)}$ and $\mathbf{V}_3^{(k)}$ define the plane of the cross section at node k); $r^{(k)}$, $k = 1,2,3$ are functions specifying the radial position of material points; $N^{(k)}$, $k = 1,2,3$ are the (quadratic) Lagrangian interpolation functions. Note that for a tube of constant cross-section:

$$r^{(k)}(\xi^3)=R_i+\frac{R_o-R_i}{2}(1+\xi^3) \tag{2}$$

R_i and R_o being the inner and outer radii of the tube.

The governing (equilibrium) equations are those implied by the principle of virtual work:

$$\int_{\Omega}\delta u_{i,j}\sigma^{ij}\,d\Omega+\int_{\Omega}\rho\,\delta\mathbf{u}\cdot\ddot{\mathbf{u}}\,d\Omega=\int_{B}\delta\mathbf{u}\cdot\mathbf{T}\,dB \tag{3}$$

in which σ^{ij} are the (contravariant) components of the Cauchy (true) stress tensor, $\delta\mathbf{u}$, $\ddot{\mathbf{u}}$ and $\mathbf{T}$ are virtual displacement, acceleration and boundary traction vectors, Ω and B are the domain and boundary in the deformed configuration, ρ is the mass density (in the deformed medium), and δu_{ij} are the covariant components of the virtual displacement gradient:

$$\delta u_{i,j}=\frac{\partial\delta\mathbf{u}}{\partial\xi^j}\cdot\mathbf{g}_i \tag{4}$$

$\mathbf{g}_i$, $i = 1, 2, 3$, being the covariant basis vectors:

$$\mathbf{g}_i=\frac{\partial\mathbf{x}}{\partial\xi^i} \tag{5}$$

The virtual displacement field in the element is obtained from Eq. 1 by differentiation and can be expressed in terms of nodal displacements $\delta\mathbf{u}^{(k)}$ and rotations $\delta\theta_1^{(k)}$, $\delta\theta_2^{(k)}$, $\delta\theta_3^{(k)}$, $k = 1, 2, 3$, with respect to the vectors $\mathbf{V}_1^{(k)}$, $\mathbf{V}_2^{(k)}$, $\mathbf{V}_3^{(k)}$:

$$\begin{aligned}\delta\mathbf{u}=\sum_{k=1}^{3}N^{(k)}(\xi^1)\Big\{&\delta\mathbf{u}^{(k)}\\&+r^{(k)}(\xi^3)\left[\left(-\delta\theta_3^{(k)}\mathbf{V}_1^{(k)}+\delta\theta_1^{(k)}\mathbf{V}_3^{(k)}\right)\sin(\xi^2)\right]\\&+r^{(k)}(\xi^3)\left[\left(-\delta\theta_1^{(k)}\mathbf{V}_2^{(k)}+\delta\theta_2^{(k)}\mathbf{V}_1^{(k)}\right)\cos(\xi^2)\right]\Big\}\end{aligned} \tag{6}$$

The internal virtual work (left-hand side of Eq. 3) can be evaluated conveniently in the undeformed domain Ω_0. Eq. 3 becomes:

$$\int_{\Omega_0}\delta u_{i,j}\tau^{ij}\,d\Omega_0+\int_{\Omega_0}\rho_0\,\delta\mathbf{u}\cdot\ddot{\mathbf{u}}\,d\Omega_0=\int_{B}\delta\mathbf{u}\cdot\mathbf{T}\,dB \tag{7}$$

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Conversion: 1m = 3.281 ft, 1N = 0.2248 lb.
KEY WORDS: Riser, dynamic, analysis, finite element, nonlinear.