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Time Variant Wave Drift Damping and Its Effect on the Response Statistics of Moored Offshore Structures

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ABSTRACT

A new method for investigating the wave drift response statistics of nonlinear compliant offshore structures is described. Compared to the typical spectral bandwidth of the slow-drift response of moored structures, the wave drift excitation forces may be considered as broad band. It is well-known that the response of dynamic systems to broad band stochastic excitation in many cases is well approximated by modelling it in terms of a multidimensional Markov process. By exploiting this observation, it is shown how the theory of Markov diffusion processes can be applied to the case of wave drift response. The power of the method developed in this paper lies in its ability to provide accurate response estimates for almost any nonlinearity. At present, the most serious drawback is the (practical) limitation to single-degree-of-freedom (SDOF) systems.

NOMENCLATURE

FPK	:	Fokker-Planck-Kolmogorov
PDF	:	Probability density function
SDE	:	Itô stochastic differential equation
SDOF	:	Single-degree-of-freedom
TLP	:	Tension leg platform
TPD	:	Transition probability density

INTRODUCTION

The traditional way of providing estimates of the response statistics of nonlinear dynamic models subjected to stochastic excitation is to carry out time domain Monte Carlo simulations. However, for the estimation of wave drift response such a procedure requires, at present, excessive computer resources, and it is not effective for prediction at low probability levels.

An alternative, and much used, method of providing estimates of the response statistics of nonlinear structures is to replace the nonlinear equations of motion by 'equivalent' linear, time-invariant ones. This is usually achieved by invoking the method of stochastic linearization (Atalik and Utku, 1976; Roberts and Spanos, 1990). Since the case of linear dynamic systems can be analyzed rather satisfactorily, this approach is clearly attractive. However, the standard way of carrying out the stochastic linearization usually admits only good estimates of second-order statistical moments (variance) of the response, and this is not enough for prediction of large and extreme responses. Recently Naess et al. (1990, 1992) proposed an extended method of stochastic linearization specifically designed for prediction of extreme responses, which seems to be of some promise.

Nevertheless, it is clearly of great practical interest if accurate general methods of wide applicability can be established that would allow estimation of response statistics, including extremes, of nonlinear dynamic systems. In this paper we shall describe a solution procedure that to a large extent has this property. It is well-known that the response of nonlinear dynamic systems to broad band random excitation can very often be accurately described by applying the theory of multidimensional Markov processes. By this, the extensive theory of Markov processes can be brought to bear on these problems. In particular, it can be shown that the probability law of response quantities can be derived through solving a partial differential equation known as the Fokker-Planck-Kolmogorov (FPK) equation (Risken, 1989; Wong and Hajek, 1985). In most cases of practical interest, this equation has to be solved numerically, but a direct solution has proved to be difficult to obtain. Here we shall develop an indirect way of solving the FPK equation. The method we shall use rests on work done in theoretical physics, where a formal solution to the FPK equation has been formulated. This kind of solution is usually known as a path integral solution. Adapting this formalism to the case of nonlinear dynamic systems has turned out to provide a powerful procedure for calculating response statistics (Naess and Johnsen, 1990).

The first attempt to apply the theory of Markov processes to investigate the second-order, slowly varying motions of moored offshore structures seems to be due to J.B. Roberts (1981). He studied the statistics of the wave drift response of a floating, nonlinearly moored structure subjected to random waves. Roberts formulated the problem in terms of a three-dimensional Markov vector process, but no attempt was made to solve the associated FPK equation. Instead the problem was simplified by assuming that the wave drift excitation forces can be modelled in terms of a white noise process when the damping is light. However, by adopting a white noise approximation to the second-order excitation forces, these forces are in effect assumed to be Gaussian, which does not appear to be appropriate in general. In particular, this procedure appears to be inadequate for the purpose of extreme response prediction.

In this paper we shall apply the path integral solution technique

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