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On the Utilization of a Block Lanczos Algorithm for the Free Vibration Analysis of Offshore Structures

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ABSTRACT

This paper examines a block type Lanczos algorithm for the free vibration analysis of offshore structures. The technique is briefly presented and then applied to the study of two structural models: a self-elevative drilling unit and a steel jacket platform. The results suggest that a large block size is not necessary for an improvement in the performance of the algorithm for current applications.

INTRODUCTION

Dynamic analysis of structures often requires the solution of the generalized eigenvalue problem:

$$\mathbf{K}\phi - \lambda\,\mathbf{M}\phi = \mathbf{0} \tag{1}$$

where **K** and **M** are, respectively, n×n stiffness and mass symmetric matrices from finite element approximations, and (λ,ϕ) is an eigenpair. Solutions of eigenproblem (Eq. 1) correspond to the undamped free vibrations of the structural system (Bathe, 1982): $\lambda = \omega^2$, with ω a free vibration frequency, and ϕ a mode shape vector. The number of equations (and also possible solutions) n is equal to is the number of degrees of freedom from the discretization process. For most analyses either the lowest λ, and corresponding ϕ, or the eigenpairs lying in a given range are required. However, even for a moderate value of n, say $n \cong 1000$, the solution of this kind of problem is a computationally intensive task. In such cases, vector iteration solution methods are usually employed.

In recent years the Lanczos algorithm (Parlett, 1980) has been widely studied and applied to the solution of eigenproblems related to free vibration and linear stability analyses. Although in some implementations the generalized eigenproblem (Eq. 1) has to be first transformed to $\mathbf{Az} - \lambda\mathbf{z} = \mathbf{0}$, where **A** is a symmetric matrix and $\lambda = 1/\omega^2$ (Chang, 1986, for example), such a transformation is actually not necessary. The method has been found to be an efficient tool for solution of large sparse generalized symmetric eigenvalue problems, as Nour-Omid et al. (1983) showed. In addition, the Lanczos algorithm can be directed to compute existing solutions close to a value σ by means of a spectral transformation (Ericsson and Ruhe, 1980). Therefore, it can search for solutions in any range of interest. Bostic and Fulton (1987), for instance, have implemented the algorithm on a parallel computer, assigning a different σ to each processor. On the other hand, the vectors generated by the algorithm have also been employed in dynamic response calculations as an alternative to modal coordinates, as suggested by Nour-Omid and Clough (1984, 1985).

Block versions of Lanczos' method are intended to improve the convergence characteristics for clustered and/or multiple eigenvalues (Golub and Underwood, 1977). Earlier versions employed a total reorthogonalization scheme in the computation of the set of Lanczos vectors to avoid a loss of orthogonality among them. However, both selective orthogonalization (Parlett and Scott, 1979) and partial reorthogonalization (Simon, 1984) used in the single Lanczos algorithm can be adapted to the block-type procedure. Therefore, the three-term recurrence, characteristic of the one-vector Lanczos process (Hughes, 1987), can be generalized to a blocked form. More recent and ingenious codes have enhanced the blocked process with a spectral transformation and an automatic definition of shifts σ, in order to speed convergence and validate the eigensolutions (Grimes et al., 1986).

This work is concerned with the utilization of a block-shifted Lanczos algorithm for the free vibration analysis of offshore structures. The algorithm we have implemented for solution of the generalized eigenproblem (Eq. 1) reflects our experience with Lanczos procedures in offshore engineering (Coutinho et al., 1987; Marques et al. 1988, 1990). In the following sections some aspects of the technique are first outlined. Then, two structural models are analyzed: a simplified model of a self-elevative drilling unit, whose free vibration frequencies are clustered, and a steel jacket platform designed to 170-m water depth. With these applications we examine the performance of the technique for different block sizes.

BLOCK LANCZOS ALGORITHM

The block Lanczos algorithm is somewhat similar to the basic algorithm (Parlett, 1980; Hughes, 1987). It generates a sequence of **M**-orthogonal blocks of vectors (or matrices) from a starting block $\mathbf{Q}_1$ as shown in Table 1.

The shift selection in step 1a will be discussed later. Parameters p and $jmax$ depend on the available memory and on some previous information from the problem to be solved. Matrices $\mathbf{Q}_j$ and $\mathbf{R}_j$ are n×p; $\mathbf{A}_j$ is p×p not exactly symmetric but only its upper triangle can be considered; and $\mathbf{B}_j$ is p×p upper triangular. The solution of the system of equations in step 2a is accomplished by a previous Gauss factorization:

$$\mathbf{K}_\sigma = \mathbf{K} - \sigma\mathbf{M} = \mathbf{LDL}^T \tag{2}$$

where **L** is a lower triangular matrix and **D** is a diagonal matrix, so that the number of negative signs in **D** is equal to the number of eigenvalues less than σ (Bathe, 1982). The **M**-orthogonal fac-

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