Bending Stiffness in a Simulation of Undersea Cable Deployment

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ABSTRACT

Bending stiffness is introduced in a three-dimensional model for submerged cable dynamics to eliminate singular behavior when cable tension becomes zero. The equations of motion are written in a local tangential-normal reference frame, and are simplified by neglecting the torsional rigidity of the cable. A centered-centered finite difference algorithm is used for the numerical simulation. The addition of bending stiffness eliminates the singularity for zero tension. However, because the bending stiffness is small, sharp gradients in the shear forces and bending moments occur at the boundaries. In this paper two sets of results are presented: configuration of an anchoring system during a steady tow, and the tension and geometry of a cable immediately following touchdown on the seafloor. The first result illustrates the discretization error caused by the sharp gradients in the curvatures and shear forces. The second result provides evidence that the bending stiffness does prevent singular behavior for negative or zero tension. It also illustrates that the contact of the cable with the seafloor only affects the cable geometry close to the seafloor.

NOMENCLATURE

J	: rotational inertial matrix
K	: matrix composed of curvatures
T	: matrix for cross product of tangent vector with force vector
Γ	: rotation matrix
Λ	: matrix premultiplier in curvature, and rotation rate expressions
Ω	: matrix composed of rotation rates
γ	: local rotation matrix
$\underline{a}$	: arbitrary vector in local coordinates
$\underline{a}_r$	: arbitrary vector in reference coordinates
$\underline{f}$	: internal force vector
$\underline{h}$	: hydrodynamic force vector
$\underline{j}$	: local current vector
$\underline{j}_r$	: current vector in reference coordinates
$\underline{k}$	: local curvature vector
$\underline{q}$	: internal moment vector
$\underline{r}$	: position vector
$\underline{u}$	: relative velocity vector
$\underline{v}$	: local velocity vector
$\underline{w}$	: weight force vector
$\underline{\chi}$	: vector of Euler angles
$\underline{\tau}$	: tangent vector
$\underline{\omega}$	: local rotational velocity vector
A	: cross sectional area
C_d	: drag coefficient spherical clump weight
C_n	: normal drag coefficient
C_t	: tangential drag coefficient
E	: Young's Modulus
G	: torsional stiffness
I	: moment of inertia

M	: mass of spherical clump weight
M_a	: mass plus added mass of spherical clump weight
S_x	: easting velocity of tow point
S_y	: northing velocity of tow point
S_z	: vertical velocity of tow point
T_a	: cable tension
W_l	: wet weight of spherical clump weight
x	: positive distance along X axis
y	: positive distance along Y axis
z	: positive distance along Z axis
d	: diameter
l_i, m_i, n_i	: direction cosines
m	: mass per unit length
m_a	: added-mass per unit length
s	: unstretched arc-length along cable
t	: time
v_p	: cable deployment rate
w_o	: wet weight per unit length
ε	: axial strain
v	: Poisson's ratio
θ, ϕ, ψ	: Euler rotation angles

INTRODUCTION

The objective of this work is the development of a robust time domain simulation of undersea cable dynamics. The simulation will be used in the prediction of cable motion, and tension during the deployment of undersea communication systems. Knowledge of cable dynamics during deployment is important. The dynamics will determine the maximum tension in the cable, and the final resting configuration of the cable on the seafloor. Predictions of cable tension are important for the obvious reason that the strength of the cable cannot be exceeded during deployment. The prediction of the cable configuration on the bottom is important for preventing the occurrence of suspensions across bottom features, and preventing the pile-up of cable around heavy electronics housings.

In a previous paper (Burgess, 1990) a centered-in-space, centered-in-time finite difference algorithm was presented for the simulation of deployment of cables with no bending stiffness.