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Structural Dynamics of Suspended Cables Supporting Arrays of Offset Bodies Part II: In-Plane Response

S.-P. Cheng and N. C. Perkins
Mechanical Engineering and Applied Mechanics
University of Michigan, Ann Arbor, Michigan, USA

ABSTRACT

In Part I of this paper, a general model is derived that governs the geometrically nonlinear, three-dimensional response of suspended cables supporting an array of offset bodies. An asymptotic form of this model is derived herein, which describes the linear response of the suspension in its equilibrium plane. The resulting model is amenable to closed-form analysis and is rich enough to capture dominant sagged cable effects, including small equilibrium cable curvature and dynamic cable tension. Free response characteristics are computed using a closed-form solution strategy based on transfer matrices. Example solutions highlight how the natural frequency spectrum, vibration mode shapes and dynamic cable tension depend on suspension symmetry.

INTRODUCTION

A general model is developed in Part I that describes the geometrically nonlinear, three-dimensional response of a cable/body suspension about a sagged equilibrium. Upon linearization, the equations governing out-of-plane response decouple from those governing in-plane response. Moreover, for sagged cable/body suspensions with small equilibrium curvature, the out-of-plane model reduces to that of a taut string supporting an array of pendula.

The present investigation of in-plane response requires a richer model for the cable/body suspension that captures the dominant sagged cable effects reviewed in Triantafyllou (1984a).These effects, which derive from cable elasticity and sag, are first noted in the small-sag model of Shea (1955) and later in Simpson (1966) and Soler (1970). A comprehensive investigation of small-sag cable theory is provided by Irvine and Caughey (1974), who show that in-plane vibration depends on a single cable parameter, λ^2, that describes the cable equilibrium tension and geometric and material properties. For horizontal suspensions, the in-plane modes are either symmetric or anti-symmetric with respect to the cable mid-span. These modes are distinguished by the fact that (anti-symmetric) symmetric modes (do not) induce dynamic cable tension and the associated natural frequencies (do not) depend on λ^2. Plotted versus λ^2, the natural frequencies of a pair of symmetric and anti-symmetric modes *crossover* (are equal) at the particular parameter values $\lambda^2 = 4j^2\pi^2$, j=1,2,..., which mark the transition of each symmetric mode from that of a taut string ($\lambda^2 \to 0$) to that of an inextensible cable or "chain" ($\lambda^2 \to \infty$).

Triantafyllou (1984b) extends the small-sag theory to include support inclination and notes that the frequency crossover phenomenon does not, in general, exist. For inclined cables, frequency crossover is replaced by frequency "avoidance" and the cable modes are "hybrid" (neither symmetric or anti-symmetric). Moreover, the hybrid modes may *all* induce dynamic cable tension. Perkins and Mote (1986) observe that frequency avoidance in the suspended cable problem represents an example of the eigenvalue *curve veering* phenomenon which occurs in diverse fields of mathematical physics. In the context of the suspended cable, they demonstrate that support inclination introduces system asymmetry, which splits the degenerate frequencies (crossover) of the symmetric (horizontal) suspension.

Qualitatively similar behavior occurs for suspensions that support attachments (e.g., arrays of buoys, weights, instrumentation, etc.) where suspension asymmetry follows from array asymmetry. Analytical studies of cables supporting single (Cheng and Perkins, 1992a) or numerous (Cheng and Perkins, 1992b) attached bodies demonstrate that frequency crossovers exist only for symmetric arrays. For asymmetric arrays, regions of frequency crossover evolve into regions of curve veering. The significance of this result is twofold. First, the mode shapes associated with the veering region are extremely sensitive to small changes in the cable parameter λ^2 (e.g., small changes in cable sag). As a result, this sensitivity would be inherited in any subsequent analysis of strumming and/or drag force distribution; see, for example, analyses described in Griffin (1985). Secondly, the vibration modes are, in general, asymmetric modes and may all induce dynamic cable tension to a degree that strongly depends on λ^2. These features have been overlooked in related numerical studies of suspensions with attached bodies (Rosenthal, 1981; Griffin and Rosenthal, 1985). The present study provides a further extension for suspensions that support arrays of offset bodies.

IN-PLANE RESPONSE MODEL

Fig. 1 of Part I illustrates a suspended cable that supports an array of l pendula. A general model describing the geometrically nonlinear, three-dimensional response of the cable/body suspension is developed in Part I, through application of Hamilton's principle (Eq. 2). In Part I, the substitution of Eqs. 3~11 into Eq. 2 leads to terms that can be ordered according to their dependence

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