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The Ill-Posed Problem of a Cable in Compression

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ABSTRACT

The linear and nonlinear dynamics of perfectly flexible cables under compression constitute an ill-posed problem. The linear problem is subject to an explosive instability, which renders any numerically obtained solution meaningless. The nonlinear problem is subject to a similar explosive instability characterized by short travelling waves of unbounded growth. The introduction of additional terms, which must include higher spatial derivatives, is essential to ensure a well-posed problem.

INTRODUCTION

In a perfectly flexible cable, tension is the only internal force, acting tangentially with respect to the instantaneous cable configuration. Since tension terms constitute the only restoring mechanism, cables under low or even zero tension have complex nonlinear response. In most applications highly taut cables are employed; recently, however, low-tension cables have become increasingly important for such applications as small tethered underwater vehicles, marine neutrally buoyant cables supporting hydrophones, fiber optic lines deployed in the ocean, towed arrays in sharp maneuvers and space tethers supporting instruments from satellites. In addition, even highly taut cables under dynamic load may lose tension, or be subjected momentarily to compression. One must then face two questions: What is the actual physical behaviour of a cable in compression; and how specific cable models behave in such situations.

In this paper we will outline all developments for two-dimensional cable dynamics, because they capture the essence of the behaviour of the cable, while the derivations are simpler. We begin by deriving the governing equations of motion along the local tangential and normal coordinates, using the unstretched lagrangian coordinate s as the spatial variable. Let u,v denote the components of the velocity vector of an element of the cable of length ds along the local tangential and normal directions respectively; T the tension force, pointing along the tangential direction $\hat{t}$; m the mass per unit length; and R_t,R_n the components of the distributed force per unit length along the local tangential and normal directions, respectively. Then we find:

$$m\left(\frac{\partial u}{\partial t}-v\frac{\partial \phi}{\partial t}\right)=\frac{\partial T}{\partial s}+R_t(1+e) \tag{1}$$

$$m\left(\frac{\partial v}{\partial t}+u\frac{\partial \phi}{\partial t}\right)=T\frac{\partial \phi}{\partial s}+R_n(1+e) \tag{2}$$

where ϕ denotes the inclination angle.

The compatibility relations are:

$$\frac{\partial u}{\partial s}-v\frac{\partial \phi}{\partial s}=\frac{\partial e}{\partial t} \tag{3}$$

$$\frac{\partial v}{\partial s}+u\frac{\partial \phi}{\partial s}=(1+e)\frac{\partial \phi}{\partial t} \tag{4}$$

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Finally a stress-strain relation is assumed, here represented by the nonlinear but memory-less function $f(e)$ of the axial strain e:

$$T=f(e) \tag{5}$$

In the sequel we will denote interchangeably the derivative of the function $\partial f/\partial e$ by f_e, or through a variable elastic stiffness denoted by $EA=f_e$.

THE HADAMARD ILL-POSED PROBLEM OF LINEAR DYNAMICS

We start by considering the transverse linear dynamics of a taut string, of length L under constant static tension T. The governing equation becomes:

$$m\frac{\partial^2 u}{\partial t^2}=T\frac{\partial^2 u}{\partial s^2} \tag{6}$$

subject to appropriate initial and boundary conditions:

$$\begin{aligned} &u(s,t=0)=h(s)\\ &\frac{\partial u}{\partial t}(s,t=0)=g(s)\\ &u(s=0,t)=0\\ &u(s=L,t)=0 \end{aligned} \tag{7}$$

where $h(s)$ and $g(s)$ are given functions, and L denotes the string length.

We consider the static tension to change very slowly, so as to affect the transverse dynamics only parametrically. We solve the equation when the tension is negative, i.e., $T=-H$, where H is a positive number.

Following Hadamard (1923), one poses three requirements for the solution of every linear problem: (1) that the solution exists; (2) that the solution be unique; and (3) that the solution be a continuous function of the initial conditions and coefficients of the problem for a time interval $0 \le t \le \tau$, where $\tau > 0$.

Eq. 6 does not satisfy the third requirement when the tension is negative, because if we expand the functions $h(s)$ and $g(s)$, which appear in the initial conditions, in terms of the eigenfunctions of the problem:

$$\begin{aligned} &h(s)=\sum_{n=1}^{\infty}h_n\phi_n(s)\\ &g(s)=\sum_{n=1}^{\infty}g_n\phi_n(s) \end{aligned} \tag{8}$$