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The Condition at Infinity for Second Order Wave Diffraction

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ABSTRACT

Based on the proper description of the condition at infinity for nonlinear surface wave diffraction in physical sense, the high order asymptotic solution as well as the inhomogeneous boundary condition at infinity for second order surface wave diffraction are obtained. This provides a complete and rational mathematical model to the problem, and clarifies the controversy arisen among a lot of works about it.

INTRODUCTION

In the context of potential flow theory, Stokes perturbation expansion provides an effective approach for the solution of mild nonlinear surface wave diffraction. In line with the expansion, the nonlinear problem of interest is decomposed into a series of sub-problems at different order, among them, the first order problem, known as the linear one, is incorporated with a homogeneous free surface boundary condition, the Sommerfeld radiation condition is accepted as the boundary condition at infinity for the diffraction potential. The mathematical model is complete for the first order problem, to which a wide variety of methods for solution have been developed, and the same solution could be found from these methods. On the contrary, for problems at second and even higher orders, the free surface condition becomes inhomogeneous. The inhomogeneous right-hand side term corresponds to the forcing due to all the quadratic contributions of first order waves on the free surface. Although the importance of evaluating the second order wave field around a complex structure is becoming more widely recognized, yet both theoretical and numerical developments of the problem have until recently been scant. Even for the simple case of a unidirectional wave propagating against a vertical circular cylinder, solutions of second order wave force evaluated by all authors in the references were controversial. The differences were primarily due to the inconsistent treatment of the second order free surface boundary conditions and improper description of boundary condition at infinity for the second order diffracted waves. Associated with this, there is the need to properly describe the boundary condition at infinity for the second order diffracted waves. The authors claim that any specification on the condition at infinity for the second order surface wave diffraction problem is the mathematical expression designating some physical feature of the problem. The various expressions of condition at infinity for second order wave diffraction designate different recognitions to the physical feature at infinity of the problem; among them misunderstandings are inevitable. Without proper specification on the physical behavior of the nonlinear wave diffraction at infinity, there would be no criterion to review on the existing different expressions of the condition at infinity for the second order diffracted waves.

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Based on the proper description of the condition at infinity for nonlinear surface wave diffraction in physical sense, the authors present in this paper a definite inhomogeneous boundary condition at infinity as well as a high order asymptotic solution in the far field for the second order surface wave diffraction problem, from which a complete and rigorous mathematical model is consequently derived.

PHYSICAL CONDITION AT INFINITY FOR THE SECOND ORDER DIFFRACTION PROBLEM

Assume the fluid is inviscid and incompressible, and the flow irrotational. The nonlinear wave diffraction against the body (e.g., offshore structure) can be defined as the following boundary-value problem. For the sake of convenience, two coordinate systems in space shall be defined: the polar coordinates $Or\theta z$ and the Cartesian coordinates $Oxyz$. The coordinate systems are defined such that their origins coincide on a point on the mean free surface. The Z axes point upwards, and radial axis as $\theta = 0°$ overlaps the x axis.

$$\nabla^2\varphi = 0 \qquad \text{(Fluid domain)} \tag{2-1}$$

$$\frac{\partial\varphi}{\partial t} + \frac{(\nabla\varphi)^2}{2} + g\eta = 0 \qquad (z = \eta) \tag{2-2}$$

$$\frac{\partial\varphi}{\partial t} + \frac{\partial\varphi}{\partial r}\frac{\partial\eta}{\partial r} + \frac{1}{r^2}\frac{\partial\varphi}{\partial\theta}\frac{\partial\eta}{\partial\theta} - \frac{\partial\varphi}{\partial z} = 0 \qquad (z = \eta) \tag{2-3}$$

$$\frac{\partial\varphi}{\partial z} = 0 \qquad (z = -d) \tag{2-4}$$

$$\frac{\partial\varphi}{\partial n} = 0 \qquad \text{(on the body surface)} \tag{2-5}$$

$$\text{Proper condition at infinity} \qquad (r \to \infty) \tag{2-6}$$

where, $\varphi(y,\theta,z;t)$ is the velocity potential function, d is the water depth, and g is the gravity acceleration. In case where the seabed