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Stress-Response of Offshore Structures by Equivalent Polynomial Expansion Techniques

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ABSTRACT

This paper concerns an investigation of the effects of nonlinearity of drag loading on offshore structures excited by 2D wave fields, where the nonlinear term in the Morison equation is replaced by an equivalent cubic expansion. The equivalent cubic expansion coefficients for the equivalent drag model are obtained using the least mean square procedure. Numerical results are given. The displacement response and the stress response processes obtained using the above loading model are compared with simulation results and those obtained from equivalent linearization of the drag term.

INTRODUCTION

The loading imposed on structural members of an offshore structure subjected to wave action represents one of the major steps in design of deepwater bottom-supported structures. The wave loading is normally estimated using the well-known Morison equation for a member with dimensions such that the presence of the member does not significantly disturb the wave field.

This paper concerns an investigation of the effects of nonlinearity of drag loading on offshore structures excited by irregular 2D wave fields, where the nonlinear term in the Morison equation is replaced by an equivalent cubic expansion. The structural system is modelled by a linear system with a finite number of degrees of freedom. A system reduction based on an eigenmode expansion is applied, where the frequency response matrix of the system is expressed in two terms, corresponding to the quasi-static contribution and the dynamic contribution, respectively. The first order wave theory is applied to relate the surface elevation with the local kinematics of water particles. The influence of the velocity of the structure is ignored in the drag term. It is assumed that the sea surface can be considered as a realization of a stationary zero-mean Gaussian process, which is also homogeneous in the horizontal space parameters. The response processes of the system are determined based on a spectral approach. The equivalent cubic expansion coefficients for the equivalent drag model are obtained using the least mean square procedure. The variance of the displacement response and the stress response processes obtained using the above loading model are compared with simulation results and with results obtained by using two different equivalent linearization methods of the drag term, namely by using the least mean square procedure and by the requirement that the variance of the original and the equivalent linear drag loading are alike. Several papers in recent years have dealt with methods for estimation of the displacement and stress response, obtained using polynomial expansion of the drag term in Morison's equation (Burrows, 1977, 1983, 1986; Bruce, 1985). However, a very important limitation of these approaches has been that they only deal with a quasi-static response, or that the structure is considered one-dimensional vertical, ignoring the horizontal spatial correlation of the wave loading.

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SHORT-TERM MODEL OF THE SEA STATES

The observed sea elevation, $\eta(\boldsymbol{x},t)$ at the fixed location $\boldsymbol{x} = (x,y)$ at a time t, can be considered as a realization of a nonstationary stochastic process, whose characteristic parameters vary slowly with time. Further, it is assumed that for short-term periods (a few hours) the sea surface $\eta(\boldsymbol{x},t)$ can be considered as a realization of a stationary stochastic process, which is also homogeneous in the horizontal space parameters. This process is assumed to be a zero-mean Gaussian process. A consequence of these simplifying assumptions is that within the short-term time scale the sea surface elevation is completely defined by the cross-covariance function $\kappa_{\eta\eta}(\Delta,x,\tau)$, defined as:

$$\kappa_{\eta\eta}(\Delta \boldsymbol{x}, r) = E\left[\eta(\boldsymbol{x},t)\eta(x+\Delta x, t+\tau)\right] \tag{1}$$

where $\Delta \boldsymbol{x} = (x_1 - x_2, y_1 - y_2)$, $\tau = t_1 - t_2$. (x_1, y_1) and (x_2, y_2) are the spatial coordinates of two points at the sea surface.

In structural analysis it may be more convenient to use spectral densities than correlation functions. Applying linear wave theory and assuming long-crested waves, the corresponding spectral density can be obtained as:

$$S_{\eta\eta}(\Delta \boldsymbol{x}, \omega) = \exp\left(-ik(\omega)(\Delta x \cos\theta + \Delta y \sin\theta)\right) S_{\eta\eta(\omega)} \tag{2}$$

where ω is the frequency (rad/sec), $\Delta x = (x_1 - x_2)$, $\Delta y = (y_1 - y_2)$. θ is the angle from the x-axis to the direction of wave propagation of the 2D sea state in a counterclockwise direction. $i = \sqrt{-1}$ and $k(\omega)$ is the wave number obtained as:

$$\omega^2 = kg \tanh(kh) \quad \omega \geq 0,\ k \geq 0 \tag{3}$$

where g is the acceleration of gravity and h is the water depth.